Cochannel Interference Reduction in Optical PPM-CDMA Systems

Hossam M. H. Shalaby, *Member, IEEE*

***Abstract*— A multiple-user interference reduction technique is proposed for optical code-division multiple-access (CDMA) systems. Data symbols from each user are encoded using a pulse-position modulation (PPM) scheme before multiplexing. Modified prime sequences are adopted as the signature codes in the multiplexing process. An interesting property of this code is the uniformity of the cross correlation among its sequences. This property is the main key in constructing the multiple-access interference canceler. In addition to its simplicity, this canceler offers a great improvement in the error probability as compared to the system without cancellation. A simple modification to this canceler that enhances its performance is proposed as well.**

***Index Terms*—Cochannel interference reduction, code division multiple access, direct-detection optical channel, optical CDMA, pulse-position modulation, spread spectrum.**

I. Introduction

THERE IS AN increasing interest in utilizing code-division multiple-access (CDMA) techniques in fiber-optic local area networks (LAN's) [1]–[16]. This is because of the wide bandwidth offered by the optical components. Both synchronous [1]–[6] and asynchronous [7]–[16] techniques have been studied in literature. Synchronous optical CDMA has some advantages over asynchronous CDMA [2]. Namely, both the possible number of subscribers and the number of simultaneous users (that can be accommodated for a given probability of error) are greater in the case of synchronous CDMA. Synchronization subsystems, however, are mandatory for the synchronous system. Thus, in high-data-rate applications where synchronization can be achieved easily, e.g., LAN's, synchronous optical CDMA stands as an attractive candidate.

Degradation in the performance of optical CDMA systems, even for ideal ones, is essentially due to the multiple-access interference. This type of interference results from the incomplete orthogonality of the used signature codes. If the receiver is able to extract an estimate for this interference, it can cancel or reduce its effect and performance improvement can be achieved.

Paper approved by J. J. O'Reilly, the Editor for Optical Communications of the IEEE Communications Society. Manuscript received August 16, 1996; revised October 15, 1997. This paper was presented in part at the IEEE Fourth International Symposium on Spread Spectrum Techniques and Applications (ISSSTA'96), Mainz, Germany, September 22–25, 1996.

The author was with the Electrical and Computer Engineering Department, International Islamic University Malaysia, Kuala Lumpur, Malaysia. He is now with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: ehshalaby@ntu.edu.sg).

Publisher Item Identifier S 0090-6778(98)04595-4.

The process of interference cancellation or reduction is not an easy task in general because it involves estimations of data symbols for each subscriber. This obviously increases the complexity of the receiver as the number of subscribers increases. Modified prime codes have an interesting property where the entire set of codes can be divided into groups. The users within one group are completely orthogonal whereas any two users from two different groups are not orthogonal. In this paper we utilize this property in developing a simple interference canceler.

To further improve the performance of the CDMA system, the data symbols are encoded using a pulse-position modulation (PPM) scheme before multiplexing. We have studied extensively the performance of optical PPM-CDMA systems without cancellation in [3]. Two main advantages of PPM-CDMA over binary on–off keying CDMA (OOK-CDMA) systems have been recited. Namely, under bit-error rate (BER) constraint, the maximum number of simultaneous users can not be increased in the case of OOK-CDMA without increasing the average power. In the case of PPM-CDMA, however, we can increase this number by increasing the pulse-position multiplicity M and preserving the average power fixed. Moreover, if we increased the average power, we still may not be able to accommodate all of the subscribers in the case of OOK. However, for PPM we can accommodate any number of users by increasing M. Of course, increasing the system complexity is the price to be paid in order to gain these advantages. In [6] we have developed three types of interference cancellation for OOK-CDMA systems under the restriction of using ideal photodetectors.

In this paper we suggest simple direct-detection optical PPM-CDMA systems with interference cancellation. We employ the modified prime sequences as our signature codes. The Poisson effects of the photodetection process is considered in our analysis. Comparisons between systems with and without interference cancellation are examined as well. From the implementation point of view, the complexity of our canceler is only sublinear in the maximum number of subscribers. Namely, the complexity of the system is proportional to the square root of the maximum number of subscribers. In fact, this canceler provides an estimation about the interference with the aid of the users sharing the same group only.

The rest of the paper is organized as follows. A system model for optical PPM-CDMA without interference cancellation is described in Section II along with a derivation for a lower bound on its BER. Section III is devoted to the description of the proposed interference canceler. An upper bound on the probability of error for this canceler is developed in this section as well. Numerical results and performance comparisons are demonstrated in Section IV. A simple modification to the above canceler is introduced in Section V. Finally, our conclusions and findings are given in Section VI.

II. Optical PPM-CDMA Without Interference Cancellation

We start by recalling some of the properties of the modified prime sequences. The interested reader may refer to [2] for further details. Let a prime number p be given. There are p^2 modified prime sequences that can be generated. Each code sequence has a weight equal to p and a length p^2. The codes are divided into p groups; each group consists of p different codes. The cross correlation (C_{mn}) between code m and code n is given by

$$C_{mn} = \begin{cases} p, & \text{if } m = n \\ 0, & \text{if } m \text{ and } n \text{ share the same group and } m \neq n \\ 1, & \text{if } m \text{ and } n \text{ are from different groups.} \end{cases} \tag{1}$$

Since we have at most p^2 sequences, the total number of subscribers is thus equal to p^2. Out of this number we assume that there are N active (simultaneous) users and the remaining $p^2 - N$ users are assumed idle. Each active user is assigned a code sequence randomly with a uniform distribution. This sequence is called the address, or signature, of the user. We define a random variable γ_n, $n \in \{1, 2, \cdots, p^2\}$ as follows:

$$\gamma_n = \begin{cases} 1, & \text{if user } n \text{ is active} \\ 0, & \text{if user } n \text{ is idle.} \end{cases}$$

Thus

$$\sum_{n=1}^{p^2} \gamma_n = N.$$

We assume for simplicity that user one is the desired user ($\gamma_1 = 1$). Let the random variable T represent the number of active users in the first group

$$T \stackrel{\text{def}}{=} \sum_{n=1}^{p} \gamma_n. \tag{2}$$

It is easy to check that the probability distribution of this random variable, given that user 1 is active, can be written as

$$P_T(t) = \frac{\binom{p^2 - p}{N - t}\binom{p - 1}{t - 1}}{\binom{p^2 - 1}{N - 1}}, \quad t \in \{t_{\min}, t_{\min} + 1, \cdots, t_{\max}\} \tag{3}$$

where

$$t_{\min} \stackrel{\text{def}}{=} \max\{N + p - p^2, 1\} \qquad t_{\max} \stackrel{\text{def}}{=} \min\{N, p\}.$$

In M-ary PPM-CDMA signaling format [3], a time frame of duration T is divided into M *disjoint* slots, each having a width $\tau = T/M$. Symbol $i \in \{0, 1, \cdots, M - 1\}$ is represented by signaling a single laser pulse of width $T_c = \tau/p^2$ at the leading edge of slot i. This pulse is further spread into p laser pulses; p is a prime number. The spreading process could be performed with the aid of a splitter, a tapped delay line, and a combiner. The width of each of the resulting pulses is also T_c. The relative positions of these pulses are determined according to the corresponding signature code. Thus, the underlined slot contains a sequence of optical pulses representing the code, and all other slots contain nothing.

Let $\mathbf{D}_n$ be a vector of length M representing the data symbol for user n. If user n wishes to send symbol $i \in \{0, 1, \cdots, M - 1\}$, then each entry in $\mathbf{D}_n$ will be equal to zero, except the ith entry will be equal to one. That is, $D_{n,i} = 1$ and $D_{n,j} = 0$ for every $j \neq i$. As explained in [3], receiver 1 correlates the compound received sequences of laser pulses, in each slot, with the corresponding signature code. This results in the collection of M photon counts. We denote by $Y_{1,i}$ the photon count collected over slot i for every $i \in \{0, 1, \cdots, M - 1\}$. Moreover, we denote by the vector $\mathbf{Y}_1$ the collection of the random variables $(Y_{1,0}, Y_{1,1}, \cdots, Y_{1,M-1})$ over all slots. Thus, $\{Y_{1,i}\}_{i=0}^{M-1}$ are independent Poisson random variables and $\mathbf{Y}_1$ is a Poisson random vector. We denote the mean of this random vector by the vector $\mathbf{Z}_1$. Whence

$$\mathbf{Z}_1 = Qp\mathbf{D}_1 + Q \sum_{n=2}^{p^2} C_{1n}\mathbf{D}_n\gamma_n$$

where Q denotes the average received photon count per pulse. The last term in the above equation is due to the interference and will be represented by the random vector κ. Thus

$$\mathbf{Z}_1 = Qp\mathbf{D}_1 + Q\kappa$$

where, according to (1)

$$\kappa \stackrel{\text{def}}{=} \sum_{n=p+1}^{p^2} \mathbf{D}_n\gamma_n.$$

In the subsequent analysis we assume equally likely data symbols. Thus, given $T = t$, it is easy to check that κ is a multinomial random vector with probability

$$P_{\kappa|T}(l_0, \cdots, l_{M-1}|t) = \frac{1}{M^{N-t}} \cdot \frac{(N-t)!}{l_0! l_1! \cdots l_{M-1}!} \tag{4}$$

where $\sum_{i=0}^{M-1} l_i = N - t$.

A. The Decision Rule

We employ the following decision rule. Symbol i is declared to be the correct one if $Y_{1,i} > Y_{1,j}$ for every $j \neq i$. The probability of bit error can, thus, be lower bounded as follows:

$$P_b = \frac{M}{2(M-1)} \sum_{t=t_{\min}}^{t_{\max}} P_E^t P_T(t)$$

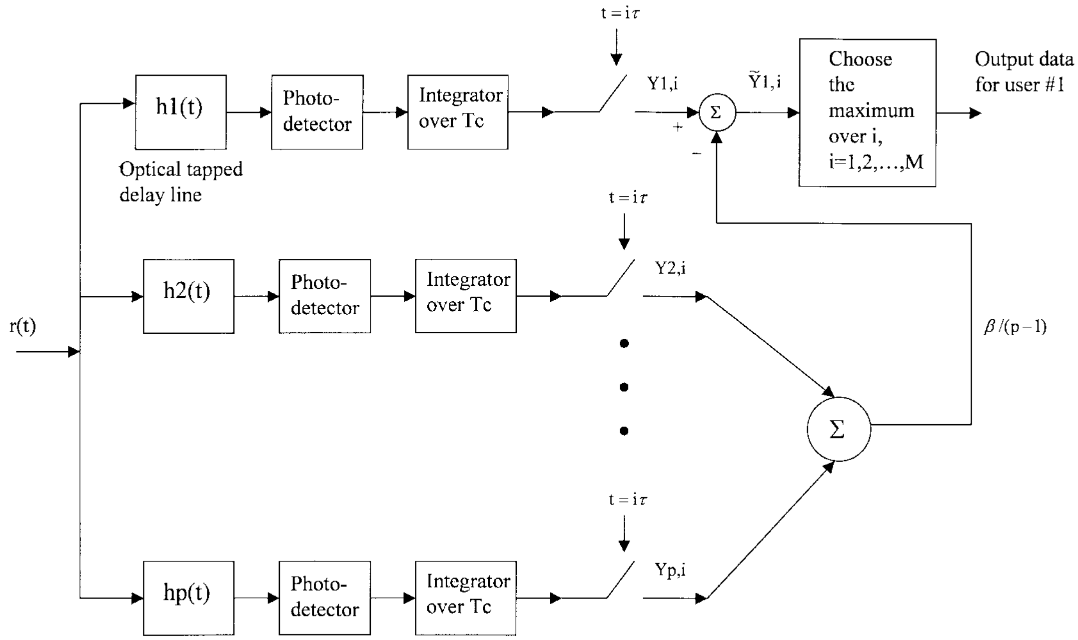

Fig. 1. Direct-detection optical PPM-CDMA system model with interference cancellation.

where

$$\begin{aligned}P_E^t =& \sum_{i=0}^{M-1} \Pr\{Y_{1,j} \geq Y_{1,i}, \text{ some } j \neq i | T=t, D_{1,i}=1\} \\ &\cdot \Pr\{D_{1,i}=1\} \\ =& \Pr\{Y_{1,j} \geq Y_{1,0}, \text{ some } j \neq 0 | T=t, D_{1,0}=1\} \\ \geq& \Pr\{Y_{1,1} \geq Y_{1,0} | T=t, D_{1,0}=1\} \\ =& \sum_{\mathbf{l}} P_{\kappa|T}(\mathbf{l}|t) \Pr\{Y_{1,1} \geq Y_{1,0} | T=t, \kappa=\mathbf{l}, D_{1,0}=1\}\end{aligned}$$

where $\mathbf{l}$ denotes the vector $(l_0, l_1, \cdots, l_{M-1})$ and

$$\begin{aligned}&\Pr\{Y_{1,1} \geq Y_{1,0} | T=t, \kappa=\mathbf{l}, D_{1,0}=1\} \\ &\quad = \sum_{y_1=0}^{\infty} e^{-Ql_1} \frac{(Ql_1)^{y_1}}{y_1!} \sum_{y_0=0}^{y_1} e^{-Q(p+l_0)} \frac{[Q(p+l_0)]^{y_0}}{y_0!}.\end{aligned}$$

It is obvious that P_E^t decreases as Q increases. Taking the limit as $Q \to \infty$, we obtain the following lower bound:

$$\begin{aligned}P_E^t \geq& \sum_{l_1=p+1}^{N-t} \binom{N-t}{l_1} \frac{1}{M^{l_1}} \left(1 - \frac{1}{M}\right)^{N-t-l_1} \\ &\cdot \sum_{l_0=0}^{\min\{l_1-p-1, N-t-l_1\}} \binom{N-t-l_1}{l_0} \frac{1}{(M-1)^{l_0}} \\ &\cdot \left(1 - \frac{1}{M-1}\right)^{N-t-l_0-l_1} + 0.5 \sum_{l_1=p}^{(N-t+p)/2} \\ &\cdot \binom{N-t}{l_1} \frac{1}{M^{l_1}} \left(1 - \frac{1}{M}\right)^{N-t-l_1} \binom{N-t-l_1}{l_1-p} \\ &\cdot \frac{1}{(M-1)^{l_1-p}} \left(1 - \frac{1}{M-1}\right)^{N-t-2l_1+p}.\end{aligned} \tag{5}$$

III. Interference Reduction in PPM-CDMA

To understand the basic idea of our interference canceler, we notice that the mean vectors $\{\mathbf{Z}_n\}_{n=2}^p$ for the photon count vectors $\{\mathbf{Y}_n\}_{n=2}^p$ collected by the users sharing the same group with user 1 are given by

$$\mathbf{Z}_n = Qp\mathbf{D}_n\gamma_n + Q\kappa.$$

That is, the average photon counts due to the interference κ are the same for all users in one group. This suggests constructing the vector $\tilde{\mathbf{Y}}_1$ as follows:

$$\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \frac{1}{p-1} \sum_{n=2}^{p} \mathbf{Y}_n. \tag{6}$$

The block diagram of this canceler is shown in Fig. 1, where in this case the decision rule for user 1 is processed aided with the entries of $\tilde{\mathbf{Y}}_1$ rather than $\mathbf{Y}_1$. Of course, we assume that all of the signature codes of the same-group users are known to each other.

We define the random vector $\mathbf{X}$ as

$$\mathbf{X} \stackrel{\text{def}}{=} \sum_{n=2}^{p} \mathbf{D}_n\gamma_n.$$

Given $T=t$, it is easy to check that $\mathbf{X}$ is again a multinomial random vector with probability

$$P_{\mathbf{X}|T}(x_0, \cdots, x_{M-1}|t) = \frac{1}{M^{t-1}} \cdot \frac{(t-1)!}{x_0!\, x_1! \cdots x_{M-1}!} \tag{7}$$

where $\sum_{i=0}^{M-1} x_i = t-1$. Moreover, define the random vector $\mathbf{R}_1$ as

$$\mathbf{R}_1 = \sum_{n=2}^{p} \mathbf{Y}_n.$$

Given $\kappa = \mathbf{l}$ and $\mathbf{X} = \mathbf{x}$, it is obvious that $\mathbf{R}_1$ is an independent Poisson random vector with mean vector

$$\mathbf{S}_1 = \sum_{n=2}^{p} \mathbf{Z}_n = Qp\mathbf{X} + Q(p-1)\kappa.$$

$\tilde{\mathbf{Y}}_1$ in (6) can now be written as

$$\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \frac{\mathbf{R}_1}{p-1}.$$

A. The Decision Rule

Similar to the case without cancellation, we adopt the following decision rule. Symbol i is declared to be the correct one if $\tilde{Y}_{1,i} > \tilde{Y}_{1,j}$ for every $j \neq i$. We derive an upper bound on the bit-error probability as follows:

$$P_b = \frac{M}{2(M-1)} \sum_{t=t_{\min}}^{t_{\max}} P_E^t P_T(t) \tag{8}$$

where

$$\begin{aligned} P_E^t =& \sum_{i=0}^{M-1} \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,i},\ \text{some } j \neq i | T=t,\ D_{1,i}=1\} \\ &\cdot \Pr\{D_{1,i}=1\} \\ =& \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,0},\ \text{some } j \neq 0 | T=t,\ D_{1,0}=1\} \\ =& \sum_{x_0} P_{X_0|T}(x_0|t) \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,0}, \\ &\quad \text{some } j \neq 0 | T=t,\ X_0=x_0,\ D_{1,0}=1\} \\ \leq& P_{X_0|T}(p-1|t) + (M-1) \sum_{x_0 \neq p-1} P_{X_0|T}(x_0|t) \\ &\cdot \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T=t,\ X_0=x_0,\ D_{1,0}=1\} \\ \leq& P_{X_0|T}(p-1|t) + (M-1) \sum_{\substack{\mathbf{l},\mathbf{x}:\\ x_0 \neq p-1}} P_{\kappa|T}(\mathbf{l}|t) P_{\mathbf{X}|T}(\mathbf{x}|t) \\ &\cdot \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T=t,\ \kappa=\mathbf{l},\ \mathbf{X}=\mathbf{x},\ D_{1,0}=1\} \\ \leq& P_{X_0|T}(p-1|t) + \sum_{\substack{\mathbf{l},\mathbf{x}:\\ x_0 \neq p-1}} P_{\kappa|T}(\mathbf{l}|t) P_{\mathbf{X}|T}(\mathbf{x}|t) \phi(t,\mathbf{l},\mathbf{x}) \end{aligned}$$

where

$$\begin{aligned} \phi(t,\mathbf{l},\mathbf{x}) \stackrel{\text{def}}{=} (M-1) \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T=t,\ \kappa=\mathbf{l}, \\ \mathbf{X}=\mathbf{x},\ D_{1,0}=1\} \end{aligned}$$

$\mathbf{l}$ denotes the vector $(l_0, l_1, \cdots, l_{M-1})$ and $\mathbf{x}$ denotes the vector $(x_0, x_1, \cdots, x_{M-1})$. Of course, $P_{X_0|T}(p-1|t) = 0$ if $t \neq p$. $\phi(t,\mathbf{l},\mathbf{x})$ can further be upper bounded as

$$\begin{aligned} \phi(t,\mathbf{l},\mathbf{x}) =& (M-1) \Pr\left\{ Y_{1,1} - \frac{R_{1,1}}{p-1} \geq Y_{1,0} - \frac{R_{1,0}}{p-1} | T=t, \right. \\ &\left. \kappa=\mathbf{l}, \mathbf{X}=\mathbf{x}, D_{1,0}=1 \right\} \\ \leq& (M-1) E\{z^{[Y_{1,1} - R_{1,1}/(p-1) - Y_{1,0} + R_{1,0}/(p-1)]} \\ &\cdot |T=t,\ \kappa=\mathbf{l},\ \mathbf{X}=\mathbf{x},\ D_{1,0}=1\} \end{aligned}$$

for every $z > 1$. We remark that the last inequality can be justified by using Chernoff bound and E denotes the expected value. Performing the expectation yields

$$\begin{aligned} \log \phi(t,\mathbf{l},\mathbf{x}) \leq& \log(M-1) - Ql_1(1-z) \\ &- Q(p+l_0)(1-z^{-1}) \\ &- Q[(p-1)l_0 + px_0)][1 - z^{1/(p-1)}] \\ &- Q[(p-1)l_1 + px_1]\{1 - z^{-[1/(p-1)]}\}. \end{aligned} \tag{9}$$

Setting $z = 1 + \delta$, where $\delta > 0$, we obtain

$$1 - z^{-1} \geq \delta - \delta^2,\ 1 - z^{1/(p-1)} \geq -\frac{\delta}{p-1}$$

$$1 - z^{-[1/(p-1)]} \geq \frac{\delta}{p-1} - \frac{p\delta^2}{2(p-1)^2}.$$

Substituting in (9) and searching for the tightest (optimum) δ yield

$$\phi(t,\mathbf{l},\mathbf{x}) \leq (M-1)\, e^{-Q\delta\mathcal{E}}$$

where

$$\mathcal{E} = 0.5\left[p + \frac{p}{p-1}(x_1 - x_0)\right]$$

$$\delta = \frac{p + \dfrac{p}{p-1}(x_1 - x_0)}{2\left[p + l_0 + \dfrac{p}{2(p-1)}\left(l_1 + \dfrac{p}{p-1}x_1\right)\right]}.$$

From the above discussion, the upper estimate on P_E^t reduces to

$$\begin{aligned} P_E^t \leq& P_{X_0|T}(x_0|t) + (M-1) \sum_{\substack{l_0,l_1,x_0,x_1:\\ x_0 \neq p-1}} \\ &\cdot P_{\kappa_0,\kappa_1|T}(l_0, l_1|t) P_{X_0,X_1|T}(x_0, x_1|t)\, e^{-Q\delta\mathcal{E}}. \end{aligned} \tag{10}$$

We notice that δ is always positive as long as $x_0 \neq p-1$. Thus, as $Q \to \infty$

$$P_E^t \to \begin{cases} 0, & \text{if } t < p \\ P_{X_0|T}(p-1|p) = \dfrac{1}{M^{p-1}}, & \text{if } t = p. \end{cases}$$

Whence if $N \geq p$, then

$$\lim_{Q\to\infty} P_b \leq \frac{\binom{p^2-p}{N-p}}{\binom{p^2-1}{N-1}} \cdot \frac{1}{2(M-1)M^{p-2}}.$$

IV. Numerical Results

Performance comparisons between optical PPM-CDMA systems with and without cancellation have been evaluated numerically with the aid of previous sections. For the system with cancellation, upper bounds on the BER have been evaluated. For the system without cancellation, however, lower bounds have been calculated. Moreover, Q is taken equal to ∞ in the case of no cancellation. Demonstrations about these comparisons are given in Figs. 2–5. In Figs. 2 and 3 the BER is plotted versus the average photons per nat μ. This is related to Q by $\mu = Qp/\log M$. It is obvious from these two figures that the error probability improves remarkably with cancellation especially for large values of

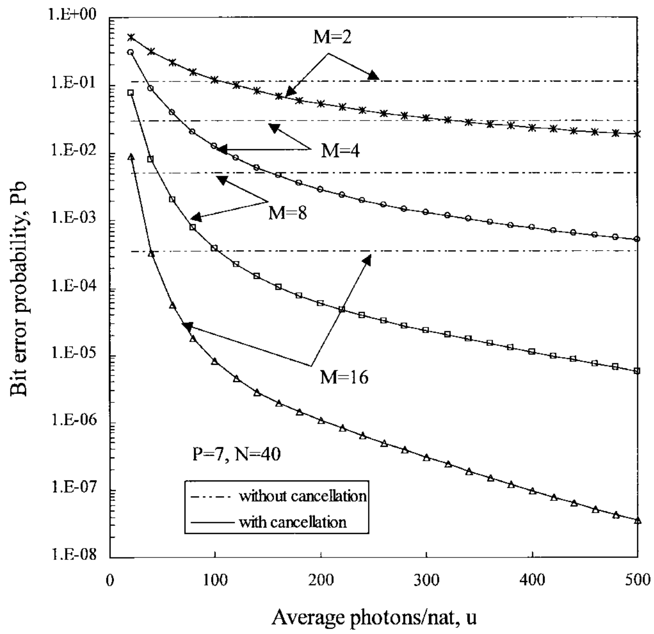

Fig. 2. A comparison between lower bounds on the BER of PPM-CDMA systems without cancellation and upper bounds on the BER of PPM-CDMA systems with cancellation for $p = 7$ and $N = 40$. The lower bounds are evaluated at $\mu = \infty$.

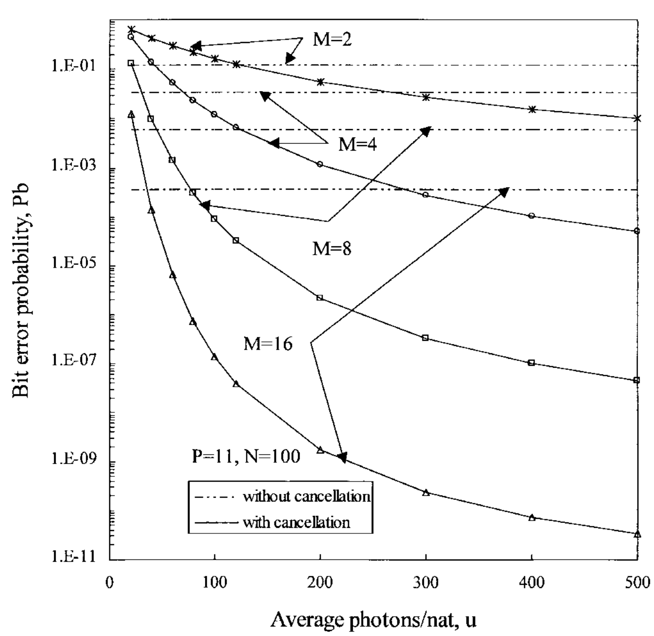

Fig. 3. A comparison between lower bounds on the BER of PPM-CDMA systems without cancellation and upper bounds on the BER of PPM-CDMA systems with cancellation for $p = 11$ and $N = 100$. The lower bounds are evaluated at $\mu = \infty$.

μ. In Fig. 4 we plot the BER versus N. It is clear that, without cancellation, the system becomes not reliable as N increases. With cancellation, however, reliability is preserved as long as μ and/or M are large enough. Fig. 5 compares the performance for the case of full load ($N = p^2$). This is plotted versus the prime number p. Again, significant improvement appears with cancellation, making the system even reliable for the full-capacity case. Conversely, with no cancellation, one can not reach the full load and retain reliable transmission.

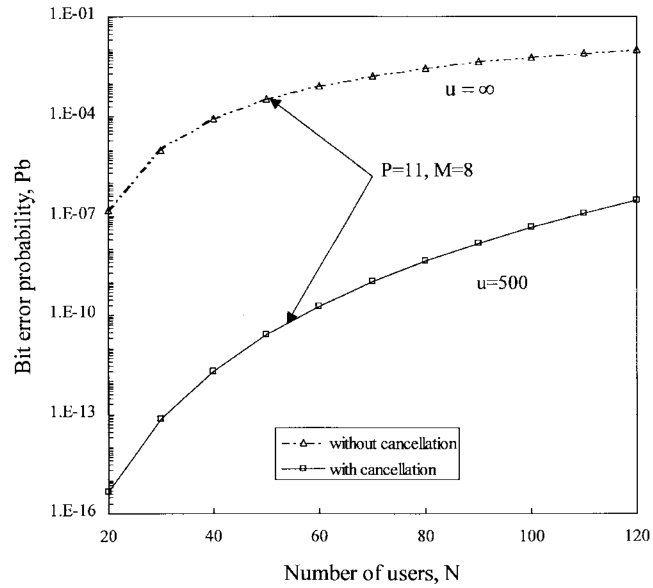

Fig. 4. PPM-CDMA BER variations versus the number of simultaneous users for $p = 11$ and $M = 8$. For the system without cancellation, a lower bound is evaluated at $\mu = \infty$. For the system with cancellation, an upper bound is evaluated at $\mu = 500$.

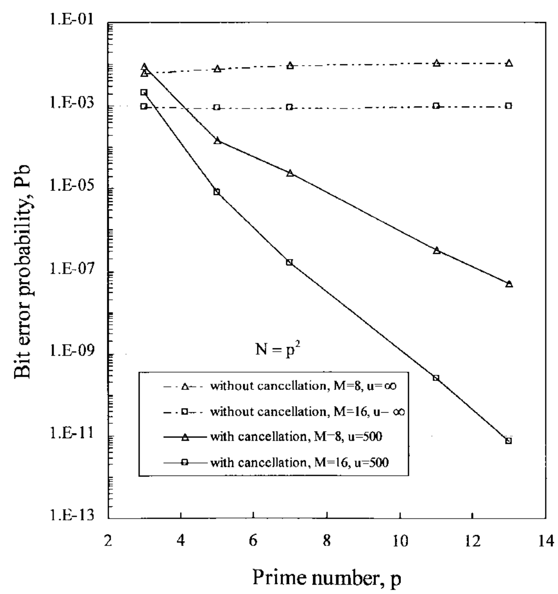

Fig. 5. A comparison of the performance of PPM-CDMA systems with and without cancellation for the case of full capacity ($N = p^2$). For the system without cancellation, a lower bound is evaluated at $\mu = \infty$. For the system with cancellation, an upper bound is evaluated at $\mu = 500$.

Furthermore, with cancellation, the performance improves as p increases so that an arbitrary small error probability can be achieved with p large enough.

V. More Interference Reduction in PPM-CDMA

In the previous canceler (called canceler 1 here) a linear combination of some users has been subtracted from the desired user in a way so as to *asymptotically* cancel the effect of the interfering vector κ. This leads to the appearance of a new interfering vector $\mathbf{X}$, which affects conversely to κ. This vector, however, *over*cancels κ, and to reduce its effect we introduce a scalar $\beta \leq 1$ in (6) as follows:

$$\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \frac{\beta}{p-1}\sum_{n=2}^{p} \mathbf{Y}_n. \tag{11}$$

Optimization over β in a way so as to enhance the cancellation effect is then needed. To simplify the analysis in this section we assume perfect optical-to-electric conversion. That is, we assume $\mathbf{Y}_n = \mathbf{Z}_n,\ n \in \{1, 2, \cdots, p^2\}$. Of course, our assumption becomes true as $Q \to \infty$. Equation (11) thus reduces to

$$\tilde{\mathbf{Y}}_1 = Qp\mathbf{D}_1 - Q\,\frac{p\beta}{p-1}\,\mathbf{X} + Q(1-\beta)\kappa.$$

We also employ here the same decision rule as canceler 1. Thus, the BER is as given in (8) but with

$$\begin{aligned} P_E^t =& \sum_{i=0}^{M-1} \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,i},\ \text{some } j \neq i | T = t,\, D_{1,i} = 1\} \\ &\cdot \Pr\{D_{1,i} = 1\} \\ =& \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,0},\ \text{some } j \neq 0 | T = t,\, D_{1,0} = 1\} \\ =& \Pr\bigg\{\frac{p\beta}{p-1}(X_0 - X_j) + (1-\beta)(\kappa_j - \kappa_0) \geq p, \\ & \text{some } j \neq 0 | T = t\bigg\}. \end{aligned} \tag{12}$$

Here three cases may arise.

Case 1—$t \leq p-1$: In this case we have

$$\begin{aligned} &\frac{p\beta}{p-1}(X_0 - X_j) + (1-\beta)(\kappa_j - \kappa_0) \\ &\quad \leq \frac{p\beta}{p-1}(t-1) + (1-\beta)(N-t) \\ &\quad \leq \frac{p\beta}{p-1}(p-2) + (1-\beta)p(p-1). \end{aligned}$$

The expression in the right-hand side can be set less than p if we choose

$$\beta > \frac{p^2 - 3p + 2}{p^2 - 3p + 3}.$$

Hence, choosing $\beta = (p^2-1)/p^2$, for example, yields $P_E^t = 0$. Also, if we choose $\beta = 1$ (canceler 1), we get same result.

Case 2—$t = p$ and $X_0 \leq p-2$: This case is similar to Case 1 and we get $P^p_{E|X_0 \leq p-2} = 0$.

Case 3—$t = p$ and $X_0 = p-1$ In this case (12) reduces to

$$P^p_{E|X_0=p-1} = \Pr\{(1-\beta)(\kappa_j - \kappa_0) \geq p(1-\beta),\ \text{some } j \neq 0 | T = p\}.$$

If $\beta = 1$ (canceler 1), then $P^p_{E|X_0=p-1} = 1$. On the other hand, if $\beta = (p^2-1)/p^2$ (canceler 2), then

$$P^p_{E|X_0=p-1} = \Pr\{\kappa_j - \kappa_0 \geq p,\ \text{some } j \neq 0 | T = p\}.$$

Combining the above results yields

$$P_b|_{\text{canceler 1}} = \frac{\binom{p^2-p}{N-p}}{\binom{p^2-1}{N-1}} \cdot \frac{1}{2(M-1)M^{p-2}}$$

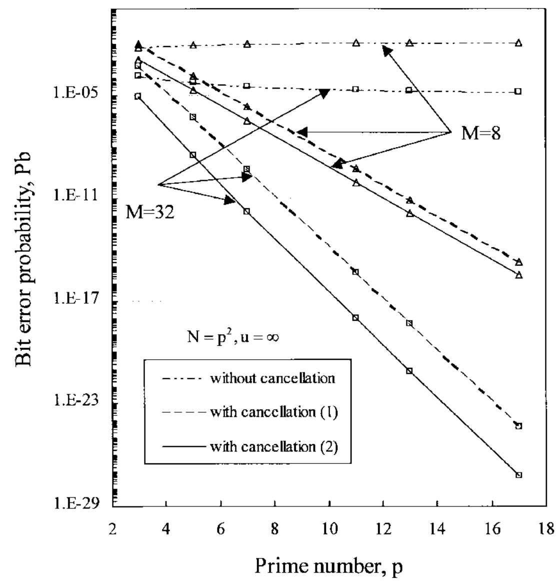

Fig. 6. A comparison of the performance of PPM-CDMA systems with and without cancellation (cancelers 1 and 2) for the case of full capacity ($N = p^2$) and $\mu = \infty$.

which is consistent to our previous result in Section III with $Q \to \infty$ and

$$P_b|_{\text{canceler 2}} = \frac{\binom{p^2-p}{N-p}}{\binom{p^2-1}{N-1}} \cdot \frac{1}{2(M-1)M^{p-2}} \Pr\{\kappa_j \geq \kappa_0 + p \ \text{some } j \neq 0 | T = p\}.$$

This can be upper bounded as

$$\begin{aligned} P_b|_{\text{canceler 2}} \leq& \frac{\binom{p^2-p}{N-p}}{\binom{p^2-1}{N-1}} \cdot \frac{1}{2M^{p-2}} \sum_{l_1=p}^{N-p} \binom{N-p}{l_1} \\ &\cdot \frac{1}{M^{l_1}}\left(1-\frac{1}{M}\right)^{N-p-l_1} \sum_{l_0=0}^{\min\{l_1-p,\, N-p-l_1\}} \\ &\cdot \binom{N-p-l_1}{l_0} \frac{1}{(M-1)^{l_0}} \\ &\cdot \left(1-\frac{1}{M-1}\right)^{N-p-l_0-l_1}. \end{aligned}$$

The above results have been evaluated numerically and the BER's have been plotted in Fig. 6. It is clear from this figure that canceler 2 performs better than canceler 1. For small values of M, the improvement over canceler 1 is not significant. However, this improvement increases as M increases.

VI. Conclusion

Two different interference cancellation techniques have been proposed for synchronous optical PPM-CDMA communication systems. Both techniques utilize the grouping property of the modified prime sequence codes. Since the signature codes of the same-group users are orthogonal, the desired subscriber collects photodetector outputs from those users and subtracts them from a scaled version of the received signal. BER's, for systems with and without cancellation, have been derived and compared. Our results demonstrate significant improvement in performance when using cancellation. Namely, we have shown that a prime sequence code always exists so that all of the subscribers are able to communicate simultaneously with arbitrary small error probability. Moreover, we have shown that the complexities of the aforementioned cancellation techniques are only sublinear in the maximum number of users. This offers a great advantage in practical implementation. Our systems stand as attractive candidates in very high-data-rate LAN's.

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Hossam M. H. Shalaby (S'83–M'83) was born in Giza, Egypt, in 1961. He received the B.S. and M.S. degrees from the University of Alexandria, Egypt, in 1983 and 1986, respectively, and the Ph.D. degree from the University of Maryland, College Park, in 1991, all in electrical engineering.

In 1991 he joined the University of Alexandria, Egypt, as an Assistant Professor of electrical engineering. He was promoted to Associate Professor in 1996. From March to April 1996, he was a Visiting Professor at the Electrical Engineering Department, Beirut Arab University, Lebanon. He has been on sabbatical leave from the University of Alexandria since September 1996. From September 1996 to January 1998 he was an Associate Professor with the Electrical and Computer Engineering Department, International Islamic University Malaysia, Kuala Lumpur. Since February 1998 he has been a Senior Lecturer with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include optical communications, optical CDMA, spread-spectrum communications, and information theory.

Dr. Shalaby received an SRC Fellowship from 1987 to 1991 (Systems Research Center, MD), and both a State Award and a Soliman Abd-El-Hay Award in 1995 (Academy of Scientific Research and Technology, Egypt). He served as a Chairman of the Student Activities Committee of Alexandria IEEE Subsection from 1995 to 1996. He has also served as a Technical Referee for the IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE TRANSACTIONS ON INFORMATION THEORY, and IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS. He is listed in the 14th edition of Marquis *Who's Who in the World*, 1997.