Chip-Level Detection in Optical Code Division Multiple Access

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***Abstract*—A new detector for optical code-division multiple-access (CDMA) communication systems is proposed. This detector is called the *chip-level* receiver. Both ON–OFF keying (OOK) and pulse-position modulation (PPM) schemes, that utilize this receiver, are investigated in this paper. For OOK, an exact bit error rate is evaluated taking into account the effect of both multiple-user interference and receiver shot noise. An upper bound on the bit error probability for pulse-position modulation (PPM)-CDMA system is derived under the above considerations. The effect of both dark current and thermal noises is neglected in our analysis. Performance comparisons between chip-level, correlation, and optimum receivers are also presented. Both correlation receivers with and without an optical hardlimiter are considered. Our results demonstrate that significant improvement in the performance is gained when using the chip-level receiver in place of the correlation one. Moreover the performance of the chip-level receiver is *asymptotically close* to the optimum one. Nevertheless, the complexity of this receiver is independent of the number of users, and therefore, much more practical than the optimum receiver.**

***Index Terms*—Code division multiple access, direct detection optical channel, ON–OFF keying, optical CDMA, pulse-position modulation, spread spectrum.**

I. Introduction

FIBER-OPTIC code-division multiple-access (CDMA) communication systems have been given an intensifying interest in the last ten years [1]–[16]. This is due to the prodigious bandwidth offered by the optical links and the extra-high optical signal processing speed bestowed by the optical components. Consequently, a superior number of simultaneous users can be accommodated in local area networks which employ optical CDMA techniques.

Most efforts in the area of direct detection optical CDMA have concentrated on either the conventional correlation receiver (with and without an optical hardlimiter) or the optimum receiver [3]–[15]. Unfortunately, simple correlation receivers perform in a faded manner as the number of users increases. This restricts full utilization of the advantages of optical CDMA. The situation for the case of optimum receivers is not so promising since computational complexity (which is exponential in the number of users) prohibits any practical realization of such systems. To get around these stipulations, Brandt-Pearce and Aazhang have suggested the multistage receiver [16]. Their results showed a significant improvement in the performance of optical CDMA systems using this detector over the correlation one. Nevertheless, their receiver complexity is only linear in the number of users.

In this paper, we propose new detection schemes for both optical direct-detection ON–OFF keying (OOK)- and pulse-position modulation (PPM)-CDMA systems. We call our new proposed systems *chip-level receivers* because as we will proclaim, they are dependent on the number of photons (optical energy) per each chip in the received frame. Our second objective of this paper is to compare the performance of the chip-level receivers with that of the correlation receivers (with and without hardlimiters) and optimum receivers.

In our performance analysis, both multiple-user interference and receiver shot noise are contemplated. The effect of both dark current and thermal noises is however abandoned since their influence on the performance is minor.

We employ optical orthogonal codes (OOC's) [1] as the signature code sequences in the analysis. We choose OOC's with periodic cross correlations and out-of-phase periodic autocorrelations that are bounded by one only. This ensures minimal interference between the users.

In order to have some insight on the results obtained we assume chip-synchronous uniformly distributed relative delays among the transmitters and perfect photon counting processes at the receivers.

Our results from the aforementioned comparisons reveal the following:

1) the performance (in terms of bit error rate) of the chip-level receiver is much more better than that of the correlation receiver and is *asymptotically* optimal (in the sense that the bit error rate approaches that of the optimum receiver as the average transmitted power increases);
2) the complexity of the chip-level receiver is independent of the number of users, and therefore, the system is much more practical than the optimum receiver.

The rest of the paper is organized as follows. Section II is devoted for a general description of both optical OOK- and PPM-CDMA transmitters and receivers. Section III is divided into four parts: In the first part we derive an expression for

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the bit error rate of OOK-CDMA correlation receiver. In the second part we define our OOK-CDMA chip-level receiver and derive an upper bound for its bit error rate. In the third part we derive a lower bound for the bit error rate of OOK-CDMA optimum receiver. In the last part of this section we derive an expression for the bit error rate of OOK-CDMA correlation receiver with an optical hardlimiter placed before the correlator. Section IV is divided into three parts. In the first and last parts, we derive lower bounds for the bit error rates of both PPM-CDMA correlation and optimum receivers, respectively. In the second part of this section, we define our PPM-CDMA chip-level receiver and derive an upper bound for its bit error rate. Section V is allocated for the presentation of some numerical results. Namely, we compare between the performances of all the aforementioned systems (under the constraint of both fixed chip time and fixed throughput) and investigate the effect of some parameters (average energy/nat, number of users, pulse-position multiplicity, etc.) on such performances. Finally, we give our conclusions in Section VI.

II. Optical CDMA System Description

In M-ary PPM signaling format each symbol is represented by a single laser pulse positioned in one of M (disjoint) possible time slots. The width of each slot is τ s. The entire symbol thus extends over a time frame of $T = M\tau$ s. This signaling format is attractive in optical communications because of its simple implementation and efficient use of the available source energy. In OOK signaling format, however, only two binary symbols are used which are represented by either the existence or nonexistence of a single laser pulse within a time slot of τ s. The width of the time frame in this case is thus $T = \tau$ s.

A typical optical CDMA communication system model is shown in Fig. 1. The transmitter is composed of N simultaneous sources of information (users). Each user produces continuous and asynchronous data symbols. The data of each user modulates a laser source using either OOK or M-ary PPM schemes. Each modulated signal is then multiplied by a periodic signature (code) sequence of length L and weight w. The chip time T_c is thus equal to τ/L. Assuming that both the chip time and throughput are held fixed, the code length is given by

$$L = \begin{cases} \dfrac{\log 2}{R_0}; & \text{for OOK} \\ \dfrac{\log M}{MR_0}; & \text{for PPM} \end{cases} \tag{1}$$

where M denotes the number of possible slots within a PPM time frame and R_0 is the throughput in nats/chip time. The output optical pulses of each multiplier (or optical CDMA encoder) are finally transmitted over an optical channel to the receiver.

At the receiving end, the received waveform is composed of the sum of N delayed and attenuated signals from each user in addition to the background noise. Each user performs its own CDMA decoding technique by multiplying the received waveform by the same underlying code sequence and then

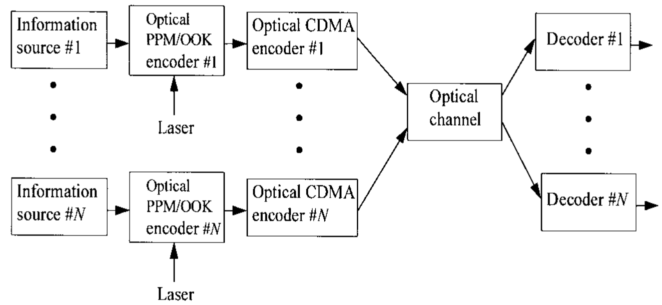

Fig. 1. Direct-detection optical OOK/PPM-CDMA system models.

converting it, using a photodetector, back to an electric signal. Finally, the output of the photodetector is forwarded to a PPM/OOK demodulator which decides on the true data.

To make full use of the vast bandwidth available to the optical network, an equivalent all-optical system can be used [2], [11]. This system is composed of optical splitters, tapped optical delay lines, and optical combiners. Of course, the bit error rate performance of both systems is the same. However, in all-optical systems we can gain much higher throughput and much larger number of users.

III. Optical OOK-CDMA

In OOK a signature sequence is transmitted (of w laser pulses) to represent data bit “1” (a mark). Data bit “0” (a space) is represented, however, by zero pulses. In OOC’s with cross correlations bounded by one, each undesired user may contribute only one pulse to this number of pulses or contribute no pulses at all (since we assume chip synchronous).

Let Y_i, $i \in \mathcal{X} = \{1, 2, \cdots, w\}$ be the photon count collected from chip number i of the mark positions. Of course Y_i is a compound Poisson random variable. Let κ_i, $i \in \mathcal{X}$ be the number of pulses (from other users) that cause interference to this chip. Further let the vector $(\kappa_1, \kappa_2, \cdots, \kappa_j)^T$ be denoted by κ^j. Hence, κ^j is a multinomial random vector with parameters $w/2L$ and $N-1$

$$\Pr\{\kappa^j = l^j\} = \frac{(N-1)!}{l_1! l_2! \cdots l_j! s_j!} \left(\frac{w}{2L}\right)^{N-1-s_j} \left(1 - j\,\frac{w}{2L}\right)^{s_j}$$

where

$$l^j = (l_1, l_2, \cdots, l_j)^T$$

and

$$s_j = N - 1 - \sum_{i=1}^{j} l_i \geq 0, \qquad j \in \mathcal{X}.$$

A. OOK-CDMA Correlation Receiver

In this receiver, the photon counts over all mark positions of the underlying code are collected to form one decision variable Y

$$Y = \sum_{i=1}^{w} Y_i.$$

The total number of interfering pulses from other users is thus

$$\kappa = \sum_{i=1}^{w} \kappa_i.$$

Of course, Y is again a compound Poisson random variable and κ is a binomial random variable with parameters $w^2/2L$ and $N-1$:

$$\Pr\{\kappa = l\} = \binom{N-1}{l}\left(\frac{w^2}{2L}\right)^l \left(1-\frac{w^2}{2L}\right)^{N-1-l}, \quad l \in \{0, 1, \cdots, N-1\}.$$

1) The Decision Rule: As usual, a threshold θ is set. If the collected photon count in one bit time (time frame) is less than this threshold, "0" is declared, otherwise "1" is declared to be sent. The probability of bit error is thus given by

$$\begin{aligned} P_b(\theta) &= \tfrac{1}{2}(P[E|0] + P[E|1]) \\ &= \tfrac{1}{2} \sum_{l=0}^{N-1} (P[E|0, \kappa = l] + P[E|1, \kappa = l]) \Pr\{\kappa = l\} \end{aligned} \tag{2}$$

where $P[E|i, \kappa = l]$ is the probability of error given that $i \in \{0, 1\}$ was sent and there are l interfering pulses with the desired user. Assuming that the average transmitted photons per chip pulse equals Q, then

$$\begin{aligned} P[E|0, \kappa = l] &= \Pr\{Y \geq \theta | 0, \kappa = l\} \\ &= \sum_{n=\theta}^{\infty} \exp[-Ql] \frac{(Ql)^n}{n!} \\ P[E|1, \kappa = l] &= \Pr\{Y < \theta | 1, \kappa = l\} \\ &= \sum_{n=0}^{\theta-1} \exp[-Q(w+l)] \frac{[Q(w+l)]^n}{n!}. \end{aligned} \tag{3}$$

B. OOK-CDMA Chip-Level Receiver

In the case of the correlation receiver, we add the photon counts over all mark positions of the underlying code to form one decision variable. There are pieces of information that may be lost after this addition. As an example, suppose that a user has sent data bit "0" and the interference pattern for that user was given by

$$\kappa_1^w = (0, 3, 3, \cdots, 3)^T.$$

From this pattern, we can decide (with small probability of error) that the user has sent a "0." However, when using a correlator, the interference variable is $\kappa = 3(w-1)$ which gives rise to the probability of a wrong decision. This example motivates us to propose the following decision rule for a new receiver, which we call the *chip-level receiver*:

1) The Decision Rule: A threshold θ is set. If the collected photon count in each mark chip of the underlying code is greater than or equal to this threshold, "1" is declared, otherwise "0" is declared to be sent. That is

$$\text{Decide} \begin{cases} 1; & \text{if } Y_i \geq \theta \qquad \forall i \in \mathcal{X}, \\ 0; & \text{otherwise.} \end{cases} \tag{4}$$

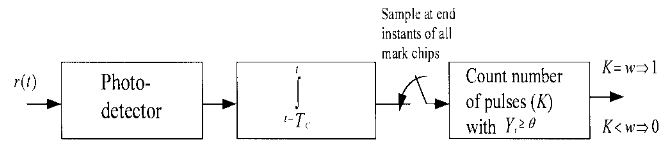

Fig. 2. Optical direct-detection OOK-CDMA chip-level receiver.

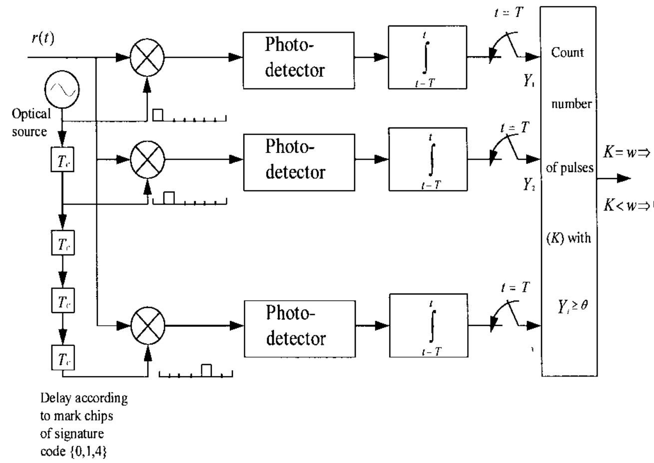

Fig. 3. An all-optical version of the OOK-CDMA chip-level receiver.

The block diagram of this receiver is shown in Fig. 2. The photodetected received signal is integrated over each chip and then sampled at the end of each mark chip. If each sample is not less than θ, a one is declared to be transmitted. Otherwise a zero is declared.

To make full use of the vast bandwidth available to the optical network, an equivalent all-optical receiver is shown in Fig. 3 where the received optical signal is sampled *optically* at the correct mark chips. Each sampled signal is then photodetected and integrated over the entire time frame ($T = LT_c$) and is further sampled *electronically* by the end of the frame. If each sampled signal is not less than θ, a one is declared to be transmitted. Otherwise a zero is declared. It is obvious that, for the second receiver, the optical signal is processed at rate $1/T_c = f_c$ whereas the electronic signal is processed at rate $1/T = f_c/L$.

In order to simplify the analysis of this receiver, we choose $\theta = 1$. Of course the bit error rate of the chip-level receiver with $\theta = 1$ forms an upper bound of the optimum chip-level receiver (with optimum θ). The probability of bit error is thus given by

$$P_b = \tfrac{1}{2}(P[E|0] + P[E|1])$$

where

$$\begin{aligned} P[E|1] &= \Pr\{Y_i = 0, \text{ some } i \in \mathcal{X}|1\} \\ &= -\sum_{i=1}^{w} (-1)^i \binom{w}{i} \Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|1\}. \end{aligned}$$

The last probability can be evaluated as

$$\begin{aligned}\Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|1\} &= \sum_{l^i} \Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|1,\, \kappa^i = l^i\} \Pr\{\kappa^i = l^i\} \\ &= \sum_{l^i} \Pr\{\kappa^i = l^i\} \prod_{j=1}^{i} \Pr\{Y_j = 0|1,\, \kappa_j = l_j\} \\ &= \sum_{l^i} \Pr\{\kappa^i = l^i\} \prod_{j=1}^{i} \exp[-Q(1 + l_j)] \\ &= \exp[-Qi] \cdot E \exp\left[-Q \sum_{j=1}^{i} \kappa_j\right] \\ &= \left[1 - i\,\frac{w}{2L} + i\,\frac{w}{2L}\, e^{-Q}\right]^{N-1} \cdot e^{-Qi}.\end{aligned}$$

Here, the second equality holds because random variables $Y_1, \cdots, Y_i$ are independent given κ^i, and E denotes the expected value over the random vector κ^i.

We can evaluate $P[E|0]$ in a similar way

$$\begin{aligned}P[E|0] &= \Pr\{Y_i \geq 1\ \forall\, i \in \mathcal{X}|0\} \\ &= 1 - \Pr\{Y_i = 0, \text{ some } i \in \mathcal{X}|0\} \\ &= 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|0\}.\end{aligned}$$

The last probability can be evaluated as

$$\begin{aligned}\Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|0\} &= \sum_{l^i} \Pr\{Y_1 = Y_2 = \cdots = Y_i = 0|0,\, \kappa^i = l^i\} \Pr\{\kappa^i = l^i\} \\ &= \sum_{l^i} \Pr\{\kappa^i = l^i\} \prod_{j=1}^{i} \Pr\{Y_j = 0|0,\, \kappa_j = l_j\} \\ &= \sum_{l^i} \Pr\{\kappa^i = l^i\} \prod_{j=1}^{i} \exp[-Q l_j] \\ &= E \exp\left[-Q \sum_{j=1}^{i} \kappa_j\right] \\ &= \left[1 - i\,\frac{w}{2L} + i\,\frac{w}{2L}\, e^{-Q}\right]^{N-1}.\end{aligned}$$

Hence, we get an expression of the overall bit error rate as follows:

$$\begin{aligned}P_b = \tfrac{1}{2}\Bigg[1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} (1 - e^{-Qi}) \\ \times \left(1 - i\,\frac{w}{2L} + i\,\frac{w}{2L}\, e^{-Q}\right)^{N-1}\Bigg]. \qquad (5)\end{aligned}$$

C. OOK-CDMA Optimum Receiver

In this subsection we derive a lower bound of the bit error rate of the optimum receiver. We assume that we got an ideal photodetector (i.e., we assume that there is no shot noise inherent in our photodetector). We obtain an optimum decision rule for that detector (which of course a lower bound to the optimum receiver) as follows. Let Z_i, $i \in \mathcal{X}$, be the number of pulses collected from chip number i of the mark positions of the underlying code. We denote the vector $Z_1, Z_2, \cdots, Z_w$ by Z^w.

We decide that data bit "1" was sent if $\Pr\{1|Z^w = z^w\} \geq Pr\{0|Z^w = z^w\}$. For equiprobable data sequences, this is equivalent to

$$\Pr\{Z^w = z^w|1\} \geq \Pr\{Z^w = z^w|0\}.$$

Notice that for $j \in \{0, 1\}$

$$\begin{aligned}\Pr\{Z^w = z^w|j\} &= \Pr\{(\kappa_1 + j, \kappa_2 + j, \cdots, \kappa_w + j)^T = z^w\} \\ &= \Pr\{\kappa^w = (z_1 - j, z_2 - j, \cdots, z_w - j)^T\} \\ &= \frac{(N-1)!}{(z_1 - j)! \cdots (z_w - j)! \left[N - 1 - \sum_{i=1}^{w} (z_i - j)\right]!} \\ &\quad \times \left(\frac{w}{2L}\right)^{\sum_{i=1}^{w} (z_i - j)} \left(1 - \frac{w^2}{2L}\right)^{N-1-\sum_{i=1}^{w} (z_i - j)}.\end{aligned}$$

Substituting in the last inequality, we can get the optimum decision rule.

1) The Decision Rule: Data bit "1" is declared to be sent if

$$\prod_{i \in \mathcal{X}} Z_i \geq \left(\frac{w/2L}{1 - w^2/2L}\right)^w \prod_{n=1}^{w} \left(N - 1 + n - \sum_{i \in \mathcal{X}} Z_i\right)$$

and otherwise "0" is declared to be sent. The probability of bit error is thus given by

$$P_b = \tfrac{1}{2}(P[E|0] + P[E|1])$$

where $P[E|1]$ and $P[E|0]$ are evaluated as follows:

$$\begin{aligned}P[E|1] &= \Pr\left\{\prod_{i \in \mathcal{X}} Z_i < \left(\frac{w/2L}{1 - w^2/2L}\right)^w \times \prod_{n=1}^{w} \left(N - 1 + n - \sum_{i \in \mathcal{X}} Z_i\right) \Bigg| 1\right\} \\ &= \Pr\left\{\prod_{i \in \mathcal{X}} (\kappa_i + 1) < \left(\frac{w/2L}{1 - w^2/2L}\right)^w \times \prod_{n=1}^{w} \left(N - 1 + n - w - \sum_{i \in \mathcal{X}} \kappa_i\right)\right\} = 0.\end{aligned}$$

The last probability does equal zero. This comes from *first* noticing that $\prod_{i \in \mathcal{X}} (\kappa_i + 1) \geq 1$ and *second* noticing that

$$\begin{aligned}\left(\frac{w/2L}{1 - w^2/2L}\right)^w \prod_{n=1}^{w} \left(N - 1 + n - w - \sum_{i \in \mathcal{X}} \kappa_i\right) &\leq \left(\frac{w/2L}{1 - w^2/2L}\right)^w \prod_{n=1}^{w} (N - 1) \\ &\leq \left[\frac{w/2L}{1 - w^2/2L} \cdot (N - 1)\right]^w.\end{aligned}$$

Since the maximum number of subscribers cannot exceed $N \leq (L-1)/w(w-1)$ and L should not be less than w^2 [1], we get

$$\left[\frac{w/2L}{1-w^2/2L}\cdot(N-1)\right]^w \leq \left[\frac{L-1}{(w-1)(2L-w^2)}\right]^w \leq \left[\frac{w^2-1}{(w-1)(2w^2-w^2)}\right]^w = \left(\frac{w+1}{w^2}\right)^w < 1$$

where we have assumed that $w \geq 2$ to justify the last inequality. Comparing the first and second notices, we conclude why $P[E|1]$ vanishes. We now evaluate $P[E|0]$ as follows:

$$\begin{aligned} P[E|0] &= \Pr\left\{\prod_{i\in\mathcal{X}} Z_i \geq \left(\frac{w/2L}{1-w^2/2L}\right)^w \times \prod_{n=1}^{w}\left(N-1+n-\sum_{i\in\mathcal{X}} Z_i\right)\Big|0\right\} \\ &= \Pr\left\{\prod_{i\in\mathcal{X}} \kappa_i \geq \left(\frac{w/2L}{1-w^2/2L}\right)^w \times \prod_{n=1}^{w}\left(N-1+n-\sum_{i\in\mathcal{X}} \kappa_i\right)\right\}. \end{aligned}$$

We show that $[(w/2L)/(1-w^2/2L)]^w \prod_{n=1}^{w}(N-1+n-\sum_{i\in\mathcal{X}}\kappa_i)$ is a positive fraction. Noticing that $0 \leq \sum_{i\in\mathcal{X}}\kappa_i \leq N-1$, we can write

$$\left(\frac{w/2L}{1-w^2/2L}\right)^w \prod_{n=1}^{w}\left(N-1+n-\sum_{i\in\mathcal{X}}\kappa_i\right) \geq \left(\frac{w/2L}{1-w^2/2L}\right)^w \prod_{n=1}^{w} n > 0.$$

On the other hand

$$\begin{aligned} \left(\frac{w/2L}{1-w^2/2L}\right)^w \prod_{n=1}^{w}\left(N-1+n-\sum_{i\in\mathcal{X}}\kappa_i\right) &\leq \left(\frac{w/2L}{1-w^2/2L}\right)^w \prod_{n=1}^{w}(N-1+n) \\ &\leq \left[\frac{w/2L}{1-w^2/2L}\cdot(N-1+w)\right]^w \\ &\leq \left[\frac{(L-1/w-1)+w^2-w}{2L-w^2}\right]^w < 1. \end{aligned}$$

The last inequality is true if

$$L+w^3-2w^2+w-1 < (2L-w^2)(w-1),$$

or ($L > w^2+1$ and $w \geq 2$). Hence

$$\begin{aligned} P[E|0] &= \Pr\left\{\prod_{i\in\mathcal{X}}\kappa_i \geq 1\right\} \\ &= \Pr\{\kappa_i \geq 1\,\forall i \in \mathcal{X}\} \\ &= 1-\Pr\{\kappa_i = 0,\ \text{some } i\in\mathcal{X}\} \\ &= 1+\sum_{i=1}^{w}(-1)^i\binom{w}{i}\Pr\{\kappa_1=\kappa_2=\cdots=\kappa_i=0\} \\ &= 1+\sum_{i=1}^{w}(-1)^i\binom{w}{i}\left(1-i\,\frac{w}{2L}\right)^{N-1}. \end{aligned}$$

Thus, the bit error rate for the ideal optimum receiver is

$$P_b = \frac{1}{2}\left[1+\sum_{i=1}^{w}(-1)^i\binom{w}{i}\left(1-i\,\frac{w}{2L}\right)^{N-1}\right]. \quad (6)$$

From the above discussion, we conclude the following two theorems.

Theorem 1: In an asynchronous optical OOK-CDMA channel employing OOC's with weight $w \geq 2$, length $L > w^2+1$, and auto- and cross-correlation constraints $\lambda \leq 1$, if the photodetector is ideal then the optimum decision rule is given by: decide data bit "1" is sent if $Z_i \geq 1\ \forall i \in \mathcal{X} = \{1, 2, \cdots, w\}$ and otherwise decide data bit "0" is sent, where $Z_i, i \in \mathcal{X}$, is the number of pulses collected from chip number i of the mark positions of the underlying code.

Proof: It is easy to check that the bit error rate of the above decision rule is equivalent to that obtained by the derived optimum rule (6). □

Theorem 2: In an asynchronous optical OOK-CDMA channel employing OOC's with weight $w \geq 2$, length $L > w^2+1$, and auto- and cross-correlation constraints $\lambda \leq 1$, if the photodetector statistics are Poisson then the chip-level decision rule is asymptotically optimum.

Proof: The proof is immediate by noticing that as $Q \to \infty$ the chip-level bit error rate, (5), approaches that of the optimum receiver, (6). □

D. OOK-CDMA Correlation Receiver with an Optical Hardlimiter

Salehi and Brackett have suggested placing an optical hardlimiter before the optical correlator in an attempt to reduce the multiuser interference power intensity [3]. We would like to emphasize that our suggested receiver is identical to the hardlimiter receiver only when using ideal photodetector. That is when ignoring the effect of the photodetector shot noise (which of course is impractical). However for practical photodetectors, the two receivers are different. To illustrate this difference we introduce the following example. Suppose that a user has sent data bit "0" and the interference pattern for that user was (similar to the previous example) given by

$$\kappa_1^w = (0, 3, 3, \cdots, 3)^T.$$

Referring to our system of Figs. 2 or 3, the photon count collected from chip number 1 after the photodetector, Y_1, is exactly "0." According to the decision rule of (4) we can decide (with zero probability of error) that the user has sent a "0." However, when using a hardlimiter before the correlator in the correlation receiver, the distribution of the average photon counts over each chip of the desired signature code immediately after the hardlimiter and before the correlator is given by

$$(0, Q, Q, \cdots, Q)^T$$

where as usual, Q denotes the average transmitted photons per chip pulse. Thus the average photon count that will be incident on the photodetector after the correlation process is $Q(w-1)$. It is thus obvious that the photon count collected from the output of the photodetector might be large and could exceed the output threshold. This would give rise to the probability of a wrong decision and a "1" would be declared with a positive probability. With the above difference in mind it should be useful to compare between these two systems. The remaining of this section is thus devoted for the derivation of an expression for the bit error rate of the hardlimiter receiver with the photodetector shot noise taken into account. The optical hardlimiter is defined as

$$g(x) = \begin{cases} Q; & \text{if } x \geq Q, \\ 0; & \text{otherwise.} \end{cases}$$

Let Z denote the photon count collected, during one bit time, from the output of the photodetector after the correlation process. The decision rule is thus: Data bit "0" is declared to be sent if Z is less than a threshold, θ, otherwise "1" is declared to be sent.

If the desired user has sent a "1," then the average incident photon count at the input of the photodetector would be equal to Qw and would be independent of the interfering users. On the other hand, if the desired user has sent a "0," then the average incident photon count would be dependent on the interfering users. Let the random variable T denote the number of interfering pulses (with the desired user) immediately after the optical hardlimiter. Of course T can take values only in the discrete set $\{0, 1, \cdots, w\}$. The average incident photon count at the input of the photodetector in this case is equal to QT. Thus the bit error rate is given by

$$\begin{aligned} P_b(\theta) &= \tfrac{1}{2}(P[E|0] + P[E|1]) \\ &= \tfrac{1}{2}\left(P[E|1] + \sum_{t=0}^{w} P[E|0, T=t] \Pr\{T=t\}\right) \end{aligned} \tag{7}$$

where $P[E|1]$ is independent of T and $P[E|0, T=t]$ is the probability of error given that a "0" has been sent and there are t interfering pulses with the desired user. Hence

$$\begin{aligned} P[E|0, T=t] &= \Pr\{Z \geq \theta | 0, T=t\} \\ &= \sum_{n=\theta}^{\infty} \exp[-Qt] \frac{(Qt)^n}{n!} \\ P[E|1] &= \Pr\{Z < \theta | 1\} \\ &= \sum_{n=0}^{\theta-1} \exp[-Qw] \frac{(Qw)^n}{n!}. \end{aligned}$$

To evaluate $\Pr\{T=t\}$ we proceed as follows:

$$\begin{aligned} \Pr\{T=t\} &= \Pr\{\kappa_{i_1} = \cdots = \kappa_{i_{w-t}} = 0, \kappa_{i_{w-t+1}} \geq 1, \cdots, \\ &\qquad \kappa_{i_w} \geq 1, i_1, \cdots, i_w \in \mathcal{X}\} \\ &= \binom{w}{t} \Pr\{\kappa_1 = \cdots = \kappa_{w-t} = 0, \kappa_{w-t+1} \geq 1, \\ &\qquad \cdots, \kappa_w \geq 1\} \\ &= \binom{w}{t} [\Pr\{\kappa_1 = \cdots = \kappa_{w-t} = 0\} \\ &\qquad - \Pr\{\kappa_1 = \cdots = \kappa_{w-t} = 0, \kappa_i = 0, \\ &\qquad \text{some } i \in \{w-t+1, \cdots, w\}] \\ &= \binom{w}{t} \Bigg[\Pr\{\kappa_1 = \cdots = \kappa_{w-t} = 0\} + \sum_{i=1}^{t} (-1)^i \\ &\qquad \times \binom{t}{i} \Pr\{\kappa_1 = \cdots = \kappa_{w-t+i} = 0\}\Bigg] \\ &= \binom{w}{t} \Bigg\{\left[1 - (w-t) \cdot \frac{w}{2L}\right]^{N-1} + \sum_{i=1}^{t} (-1)^i \\ &\qquad \times \binom{t}{i} \left[1 - (w-t+i) \cdot \frac{w}{2L}\right]^{N-1}\Bigg\} \\ &= \binom{w}{t} \sum_{i=0}^{t} (-1)^i \binom{t}{i} \left[1 - (w-t+i) \cdot \frac{w}{2L}\right]^{N-1}. \end{aligned}$$

IV. Optical PPM-CDMA

In M-ary PPM symbol $j \in \mathcal{M} =\in \{0, 1, \cdots, M-1\}$ is represented by transmitting a signature sequence within slot number j. All other slots contain, however, zero pulses. Let Y_{ij}, $i \in \mathcal{X} = \{1, 2, \cdots, w\}$, $j \in \mathcal{M} = \{0, 1, \cdots, M-1\}$, be the photon count collected from chip number i of the mark positions of slot number j. Y_{ij} is a compound Poisson random variable. Let κ_{ij}, $i \in \mathcal{X}$, $j \in \mathcal{M}$ be the number of pulses (from other users) that cause interference to chip i of slot j. Further let the vector $(\kappa_{1j}, \kappa_{2j}, \cdots, \kappa_{nj})^T$ be denoted by κ_j^n. The supervector $(\kappa_0^n, \kappa_1^n, \cdots, \kappa_m^n)^T$, $n \in \mathcal{X}$, $m \in \mathcal{M}$, will be denoted by κ^{nm}. Assuming frame-level synchronization among the transmitters, it is easy to check that κ^{nm} is a multinomial random vector with parameters w/ML and $N-1$

$$\begin{aligned} \Pr\{\kappa^{nm} = l^{nm}\} &= \frac{(N-1)!}{l_{10}! l_{20}! \cdots l_{nm}! s_{nm}!} \left(\frac{w}{ML}\right)^{N-1-s_{nm}} \\ &\quad \times \left[1 - n(m+1) \frac{w}{ML}\right]^{s_{nm}} \end{aligned}$$

where

$$\begin{aligned} l^{nm} &= (l_0^n, l_1^n, \cdots, l_m^n)^T \\ l_j^n &= (l_{1j}, l_{2j}, \cdots, l_{nj})^T \end{aligned}$$

and

$$s_{nm} = N - 1 - \sum_{j=0}^{m} \sum_{i=1}^{n} l_{ij}, \qquad n \in \mathcal{X}, m \in \mathcal{M}.$$

A. PPM-CDMA Correlation Receiver

In this receiver, the photon counts over all mark positions (of the underlying code) in each slot are collected to form decision variables Y_j, $j \in \mathcal{M}$

$$Y_j = \sum_{i=1}^{w} Y_{ij}.$$

The total number of interfering pulses (from other users) to slot j is thus

$$\kappa_j = \sum_{i=1}^{w} \kappa_{ij}.$$

Hence, κ_j is a binomial random variable with parameters w^2/ML and $N-1$.

1) The Decision Rule: Symbol "m" is declared to be transmitted if there exists $m \in \mathcal{M}$ with $Y_m > Y_j$ for every $j \neq m$; an incorrect decision is otherwise declared. We now provide a lower bound on the probability of word error P_E. The bit error rate P_b is related to P_E by the well-known formula $P_b = [(M/2)/(M-1)]P_E$

$$P_E = \sum_{j=0}^{M-1} P[E|j] \Pr\{j\}$$

where $\Pr\{j\} = 1/M$ in the case of equally likely data. It is easy to check that $P[E|j]$ is independent of j. Hence

$$\begin{aligned} P_E &= P[E|0] = \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0|0\} \\ &\geq (M-1) \Pr\{Y_1 \geq Y_0|0\} \\ &\quad - \binom{M-1}{2} \Pr\{Y_1 \geq Y_0, Y_2 \geq Y_0|0\}. \end{aligned}$$

The first probability is lower bounded as follows:

$$\begin{aligned} \Pr\{Y_1 \geq Y_0|0\} &= \sum_{l_0, l_1} \Pr\{\kappa_0 = l_0, \kappa_1 = l_1\} \\ &\quad \times \Pr\{Y_1 \geq Y_0|0, \kappa_0 = l_0, \kappa_1 = l_1\} \\ &\geq \sum_{l_1} \Pr\{\kappa_0 = 0, \kappa_1 = l_1\} \\ &\quad \times \Pr\{Y_1 \geq Y_0|0, \kappa_0 = 0, \kappa_1 = l_1\} \end{aligned}$$

where

$$\begin{aligned} &\Pr\{\kappa_0 = 0, \kappa_1 = l_1\} \\ &\quad = \binom{N-1}{l_1} \left(\frac{w^2}{ML}\right)^{l_1} \left(1 - 2\,\frac{w^2}{ML}\right)^{N-1-l_1} \end{aligned}$$

and

$$\begin{aligned} &\Pr\{Y_1 \geq Y_0|0, \kappa_0 = 0, \kappa_1 = l_1\} \\ &\quad = \sum_{y_1=0}^{\infty} e^{-Ql_1} \frac{(Ql_1)^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw} \frac{(Qw)^{y_0}}{y_0!}. \end{aligned}$$

The second probability can be upper bounded as follows:

$$\begin{aligned} &\Pr\{Y_1 \geq Y_0, Y_2 \geq Y_0|0\} \\ &\quad \leq \Pr\{Y_1 + Y_2 \geq 2Y_0|0\} = \Pr\{Y' \geq 2Y_0|0\} \\ &\quad = \sum_{l_0, l'} \Pr\{\kappa_0 = l_0, \kappa' = l'\} \\ &\qquad \times \Pr\{Y' \geq 2Y_0|0, \kappa_0 = l_0, \kappa' = l'\} \\ &\quad \leq \sum_{l'} \Pr\{\kappa' = l'\} \Pr\{Y' \geq 2Y_0|0, \kappa_0 = 0, \kappa' = l'\} \end{aligned}$$

where $Y' = Y_1 + Y_2$, $\kappa' = \kappa_1 + \kappa_2$, and

$$\Pr\{\kappa' = l'\} = \binom{N-1}{l'} \left(2\,\frac{w^2}{ML}\right)^{l'} \left(1 - 2\,\frac{w^2}{ML}\right)^{N-1-l'}.$$

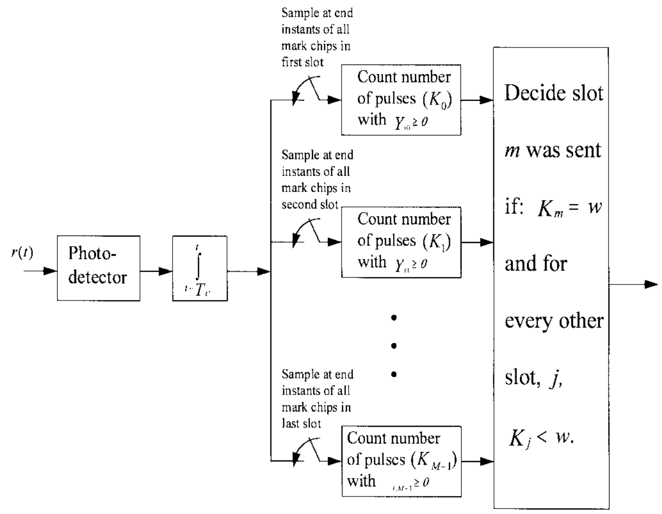

Fig. 4. Optical direct-detection PPM-CDMA chip-level receiver.

Making use of Chernoff bound, we get for any $z \geq 1$

$$\begin{aligned} &\Pr\{Y' \geq 2Y_0|0, \kappa_0 = 0, \kappa' = l'\} \\ &\quad \leq E[z^{Y'-2Y_0}|0, \kappa_0 = 0, \kappa' = l'] \\ &\quad = e^{Ql'(z-1)} \cdot e^{-Qw(1-z^{-2})}. \end{aligned}$$

Choosing a value of z so as to minimize the right-hand side (RHS) yields $z_0 = (2w/l')^{1/3}$. Thus

$$\begin{aligned} &\Pr\{Y' \geq 2Y_0|0, \kappa_0 = 0, \kappa' = l'\} \\ &\quad \leq \begin{cases} \exp[Ql'(z_0 - 1) - Qw(1 - z_0^{-2})]; & \text{if } l' < 2w, \\ 1; & \text{otherwise.} \end{cases} \end{aligned}$$

B. PPM-CDMA Chip-Level Receiver

In this section, we suggest a chip-level detector for optical PPM-CDMA systems. This receiver is correspondent to the OOK chip-level receiver that we have proposed in Section III-B.

1) The Decision Rule: Symbol "m" is declared to be transmitted if there exists $m \in \mathcal{M}$ such that

$$(\forall i \in \mathcal{X}) \qquad Y_{im} \geq 1$$

and

$$(\forall j \in \mathcal{M}, j \neq m) \qquad Y_{ij} = 0, \text{ some } i \in \mathcal{X}.$$

Otherwise an incorrect decision is declared. The block diagram of this receiver is shown in Fig. 4. The photodetected received signal is integrated over each chip and then sampled at the end of each mark chip of each slot in the time frame. If each sample is not less than one within a certain slot m and there is at least one sample that equals zero in every other slot, then m is declared to be transmitted. Otherwise an incorrect decision is declared.

To make full use of the vast bandwidth available to the optical network, an equivalent all-optical receiver can be deduced in a similar way to what we have done in the OOK case.

The probability of a word error in this case can be shown to be given by

$$\begin{aligned} P_E &= P[E|0] \\ &= \Pr\{Y_{i0}=0,\ \text{some } i\in\mathcal{X} \\ &\qquad \text{or } Y_{ij}\ge 1\,\forall\, i\in\mathcal{X},\ \text{some } j\ne 0|0\} \\ &\le \Pr\{Y_{i0}=0,\ \text{some } i\in\mathcal{X}|0\} + (M-1) \\ &\quad \times \Pr\{Y_{i1}\ge 1\,\forall\, i\in\mathcal{X}|0\}. \end{aligned}$$

The first probability is evaluated in a similar way to what we have done in OOK

$$\begin{aligned} &\Pr\{Y_{i0}=0,\ \text{some } i\in\mathcal{X}|0\} \\ &= -\sum_{i=1}^{w}(-1)^i\binom{w}{i}\Pr\{Y_{10}=Y_{20}=\cdots=Y_{i0}=0|0\} \\ &= -\sum_{i=1}^{w}(-1)^i\binom{w}{i}\left[1-i\,\frac{w}{ML}+i\,\frac{w}{ML}\,e^{-Q}\right]^{N-1}\cdot e^{-Qi}. \end{aligned}$$

The second probability is upper bounded as follows:

$$\begin{aligned} &\Pr\{Y_{i1}\ge 1\,\forall\, i\in\mathcal{X}|0\} \\ &\quad = \Pr\{Y_{i1}\ge 1,\ \kappa_{i1}\ge 1\,\forall\, i\in\mathcal{X}|0\} \\ &\qquad + \Pr\{Y_{i1}\ge 1,\ \forall\, i\in\mathcal{X},\ \kappa_{n1}=0,\ \text{some } n\in\mathcal{X}|0\} \\ &\quad \le \Pr\{\kappa_{i1}\ge 1\,\forall\, i\in\mathcal{X}|0\} + \text{zero} \\ &\quad = 1+\sum_{i=1}^{w}(-1)^i\binom{w}{i}\left[1-i\,\frac{w}{ML}\right]^{N-1}. \end{aligned}$$

The zero after the first inequality is due to the fact that given a "0" was sent, if $\kappa_{n1}=0$ then Y_{n1} should be zero as well.

C. PPM-CDMA Optimum Receiver

Assuming an ideal photodector we derive an optimum decision rule as follows. Let Z_{ij}, $i\in\mathcal{X}$, $j\in\mathcal{M}$, be the number of pulses collected from chip number i of the mark positions (of the underlying code) of slot number j. We denote the vector $(Z_{1j}, Z_{2j}, \cdots, Z_{wj})^T$ by Z_j^w and denote the super-vector $(Z_0^w, Z_1^w, \cdots, Z_{M-1}^w)^T$ by Z^w.

We decide that data symbol "m" was sent if for any $j\in\mathcal{M}$ and $j\ne m$ $\Pr\{m|Z^w=z^w\} > \Pr\{j|Z^w=z^w\}$. For equiprobable data symbols, this is equivalent to

$$\Pr\{Z^w=z^w|m\} > \Pr\{Z^w=z^w|j\}.$$

Notice that for any $j\in\mathcal{M}$

$$\begin{aligned} &\Pr\{Z^w=z^w|j\} \\ &\quad = \Pr\{\kappa^w=z^w,\cdots,\kappa_{j-1}^w=z_{j-1}^w, \\ &\qquad (\kappa_{1j}+1,\kappa_{2j}+1,\cdots,\kappa_{wj}+1)^T=z_j^w, \\ &\qquad \kappa_{j+1}^w=z_{j+1}^w,\cdots,\kappa_{M-1}^w=z_{M-1}^w\} \\ &\quad = \Pr\{\kappa^w=z^w,\cdots,\kappa_{j-1}^w=z_{j-1}^w, \\ &\qquad \kappa_j^w=(z_{1j}-1,z_{2j}-1,\cdots,z_{wj}-1)^T, \\ &\qquad \kappa_{j+1}^w=z_{j+1}^w,\cdots,\kappa_{M-1}^w=z_{M-1}^w\}. \end{aligned}$$

Noticing that $\kappa^{w(M-1)}$ has a multinomial distribution and substituting in the last inequality, we can get the optimum decision rule.

1) The Decision Rule: Data symbol "m" is declared to be sent if for any $j\in\mathcal{M}$ and $j\ne m$

$$\prod_{i\in\mathcal{X}} Z_{im} > \prod_{i\in\mathcal{X}} Z_{ij}.$$

An incorrect decision is otherwise declared. The probability of a word error is thus given by

$$\begin{aligned} P_E = P[E|0] &= \Pr\left\{\prod_{i\in\mathcal{X}} Z_{i0}\le\prod_{i\in\mathcal{X}} Z_{ij},\ \text{some } j\ne 0|0\right\} \\ &= \Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{ij},\ \text{some } j\ne 0\right\} \\ &\ge (M-1)\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i1}\right\} \\ &\quad -\binom{M-1}{2}\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i1}\right. \\ &\qquad\qquad \left.\text{and } \prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i2}\right\}. \end{aligned}$$

The first probability can be lower bounded as

$$\begin{aligned} &\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i1}\right\} \\ &\quad \ge \Pr\left\{\kappa_{10}=\kappa_{20}=\cdots=\kappa_{w0}=0,\ \prod_{i\in\mathcal{X}}\kappa_{i1}\ge 1\right\} \\ &\quad = \Pr\{\kappa_{i0}=0,\ \kappa_{i1}\ge 1\,\forall\, i\in\mathcal{X}\} \\ &\quad = \Pr\{\kappa_{i0}=0,\ \forall\, i\in\mathcal{X}\} \\ &\qquad - \Pr\{\kappa_{i0}=0\,\forall\, i\in\mathcal{X},\ \kappa_{n1}=0,\ \text{some } n\in\mathcal{X}\} \\ &\quad = \sum_{n=0}^{w}(-1)^n\binom{w}{n} \\ &\qquad \times\Pr\{\kappa_{i0}=0\,\forall\, i\in\mathcal{X},\ \kappa_{11}=\kappa_{21}=\cdots=\kappa_{n1}=0\} \\ &\quad = \sum_{i=0}^{w}(-1)^i\binom{w}{i}\left[1-(i+w)\,\frac{w}{ML}\right]^{N-1}. \end{aligned}$$

The second probability can be upper bounded as

$$\begin{aligned} &\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i1}\ \text{and}\ \prod_{i\in\mathcal{X}}(\kappa_{i0}+1)\le\prod_{i\in\mathcal{X}}\kappa_{i2}\right\} \\ &\quad \le \Pr\left\{\prod_{i\in\mathcal{X}}\kappa_{i1}\ge 1\ \text{and}\ \prod_{i\in\mathcal{X}}\kappa_{i2}\ge 1\right\} \\ &\quad = \Pr\{\kappa_{i1}\ge 1\ \text{and}\ \kappa_{i2}\ge 1\,\forall\, i\in\mathcal{X}\} \\ &\quad \le \Pr\left\{\sum_{i\in\mathcal{X}}\kappa_{i1}+\sum_{i\in\mathcal{X}}\kappa_{i2}\ge 2w\right\}\le\left[\frac{(N-1)we}{ML}\right]^{2w}. \end{aligned}$$

The last inequality is justified by using Chernoff bound [11], [12].

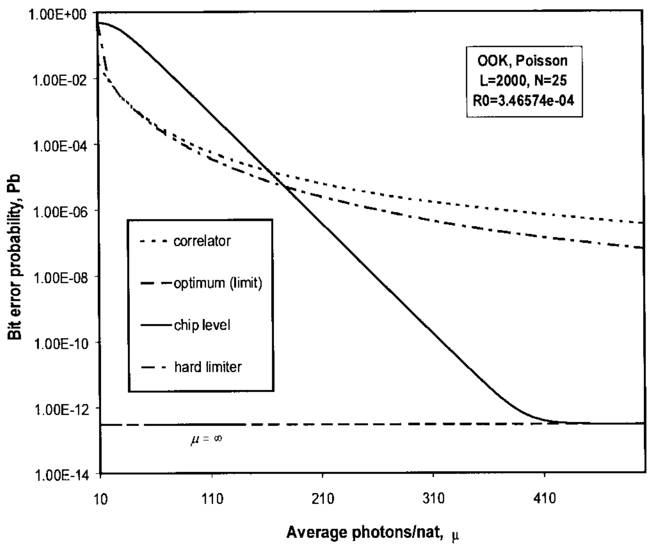

Fig. 5. A comparison between the bit error rate of correlation receiver (with and without an optical hardlimiter) and chip-level receiver for the case of OOK-CDMA systems with $L = 2000$ and $N = 25$.

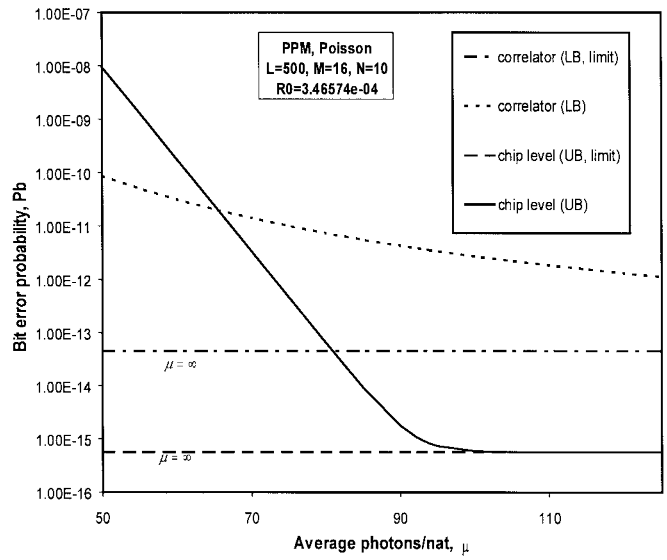

Fig. 6. A comparison between lower bounds on BER of correlation receiver and upper bounds on BER of chip-level receiver for the case of PPM-CDMA systems with $M = 16$, $L = 500$, and $N = 10$.

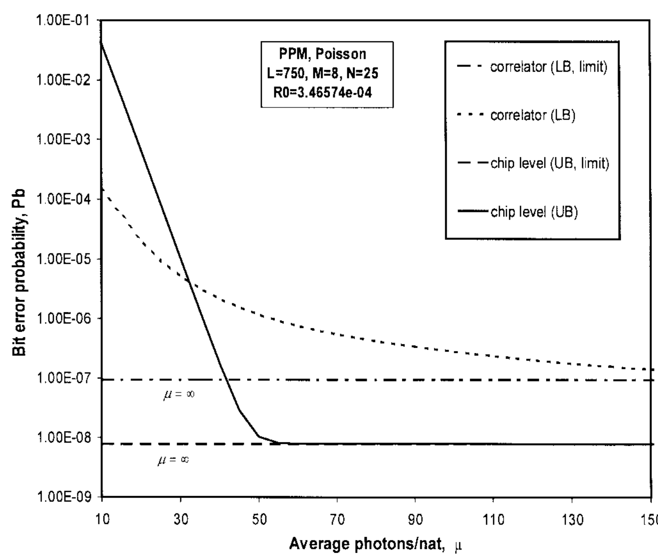

Fig. 7. A comparison between lower bounds on BER of correlation receiver and upper bounds on BER of chip-level receiver for the case of PPM-CDMA systems with $M = 8$, $L = 750$, and $N = 25$.

V. Numerical Results

Given the number of users N and the code length L, we choose the code weight w, in all our numerical calculations, to be the maximum weight that satisfies the relation [1]

$$N \leq \frac{L-1}{w(w-1)}.$$

Assuming that the average transmitted photons per nat μ is held fixed, the average transmitted photons per chip pulse Q is thus given by

$$Q = \begin{cases} \dfrac{\mu \log 2}{w}; & \text{for OOK} \\ \dfrac{\mu \log M}{w}; & \text{for PPM.} \end{cases}$$

A. Optical OOK-CDMA

The optimum thresholds which minimize the bit error rates in (2) and (7) have been evaluated numerically for $L = 2000$ and $N = 25$. The minimum bit error rates $P_b = \min_\theta P_b(\theta)$ for the correlation receiver (with and without an optical hardlimiter) are plotted in Fig. 5 for different values of the average photons per nat μ. The chip-level bit error rate is also plotted on the same graph for same parameters. Further the bit error rate limit (as $\mu \rightarrow \infty$) for the optimum receiver is superimposed on the same graph. We have to remark that both chip-level and optimum bit error rates are identical for $\mu \rightarrow \infty$ (cf. Theorem 2). The superiority of the chip-level receiver over the correlation receiver, even with the hardlimiter, is obvious from the figure. For example, if the bit error rate is required not to exceed and 5×10^{-7} we need (at least) $\mu = 454$ for the correlation receiver and $\mu = 304$ for the correlation receiver with a hardlimiter, whereas $\mu = 208$ for the chip-level receiver. This indicates that more than 54 and 31% save in energy are gained when using the chip-level receiver in place of the correlation without and with an optical hardlimiter, respectively. Furthermore, if $\mu = 454$ is used for chip-level detector we get a bit error rate of 3.04×10^{-13} which is so close to the lower bound of the optimum bit error rate of 3.01×10^{-13}.

It should be emphasized that we are comparing a suboptimal chip-level receiver's threshold ($\theta = 1$) with optimal correlation receivers' thresholds, which adds to the advantages of chip-level detection.

B. Optical PPM-CDMA

Performance comparisons between chip-level and correlation receivers for PPM-CDMA systems are shown in Figs. 6 and 7. The advantage of using chip-level decision rule is obvious; and similar conclusions to OOK-CDMA systems can be drawn from these figures.

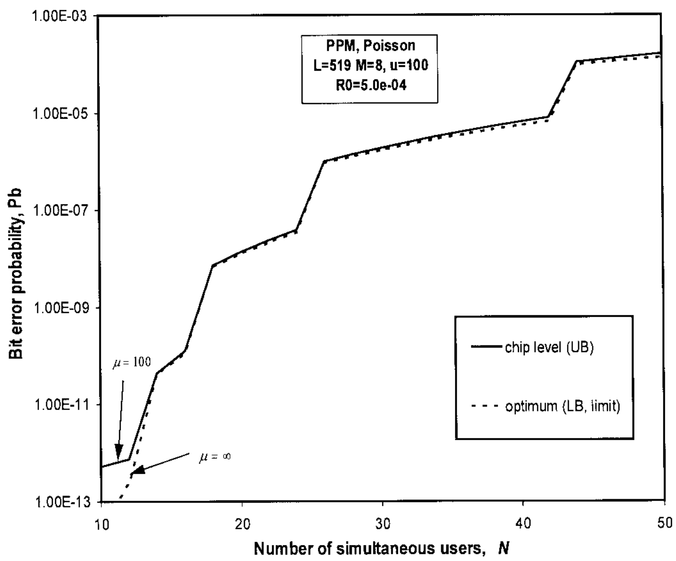

Fig. 8. A comparison between a limiting ($\mu = \infty$) lower bound on BER of the optimum receiver and an upper bound on BER of chip-level receiver (with $\mu = 100$) for the case of PPM-CDMA systems with $M = 8$ and $L = 519$ versus the number of users.

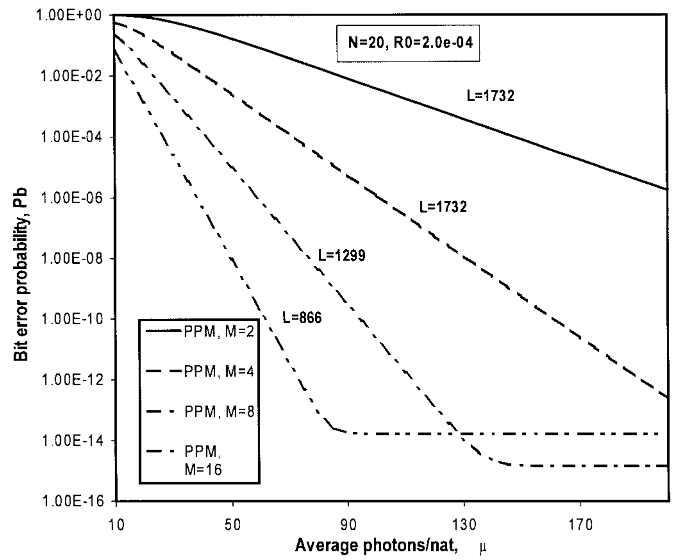

Fig. 9. A comparison between the bit error rates of PPM-CDMA chip-level receivers for different values of M, under the constraints of fixed throughput $R_0 = 2.0e-04$ nats/chip time, chipwidth T_c, and number of users $N = 20$.

A comparison between the chip-level receiver with the optimum receiver in terms of bit error rates is illustrated in Fig. 8. The throughput is fixed at $R_0 = 5.0 \times 10^{-4}$ nats/chip time. The upper bound of the chip-level bit error rate (with $\mu = 100$) has been plotted along with the lower bound of the optimum bit error rate (with $\mu = \infty$) versus the number of users N. It is too hard to distinguish between the two plots which demonstrates that our detector is really close to optimum.

Finally a comparison between the bit error rates for PPM-CDMA systems with different values of M is presented in Fig. 9 (with $R_0 = 2.0 \times 10^{-4}$ and $N = 20$). Different values of code lengths have been calculated (as M varies) so as to hold the throughput fixed. For μ small enough, the performance improves as M increases. As μ increases the bit error rates of systems with large M start to saturate a little bit early. The reason is that the larger the value of M the smaller the value of L, which causes the hit probability due to multiple-access interference to increase; and in turn a worse bit error rate and hence early saturation. The effect of multiple-access interference is, however, negligible for small values of μ which interprets the improvement of the bit error rates for systems with large M. Thus, for systems with limited energy, PPM-CDMA schemes with large M are more suitable.

C. Both Optical OOK- and PPM-CDMA

A comparison between both chip-level OOK- and PPM-CDMA bit error rates for fixed throughput, chip time, and number of users can be extracted from Figs. 5 and 7. For small values of μ PPM performs better than OOK. For example if the bit error rate is required not to exceed 10^{-8}, an average energy per nat of $\mu = 51$ is required for PPM, whereas $\mu = 259$ for OOK. That is more than 80% save in energy is gained when using PPM. However, the bit error rates of PPM systems saturate earlier than the bit error rates of OOK which continue to decrease below that of PPM. We conclude that PPM is better than OOK for systems with limited energy. However, OOK is better if the energy is not the constrained factor.

VI. Concluding Remarks

New detectors for direct-detection optical OOK- and PPM-CDMA communication systems have been proposed. These detectors have been termed chip-level receivers. Performance comparisons between chip-level receivers and both traditional correlation and optimum receivers have been presented. Optical orthogonal codes, with cross correlations bounded by one, have been considered as the signature code sequences in our systems. The Poisson shot noise model has been assumed for the receiver photodetectors. Upper bounds on the probability of error have been derived for the chip-level receivers. Nevertheless, lower bounds have been obtained for all other receivers. Numerical results have been evaluated under the restriction of both fixed chip time and fixed throughput. We can thus extract the following concluding remarks.

1) The performance of chip-level receivers is superior to that of correlation receivers and is *asymptotically* close (or equal) to that of optimum receivers.
2) The complexity of chip-level receivers is independent of the number of users, and therefore the system is much more practical than the optimum receiver.
3) Under fixed photon energy per information nat, the performance of chip-level PPM-CDMA receivers improves significantly (as expected) as M increases. However, the bit error rates saturate as μ increases; and the start instants of saturation decrease as M increases. Thus, depending on the system's energy restriction, proper values of M should be chosen from curves like that of Fig. 9.

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