On the Cutoff Rate of a Variable-Bit-Rate (VBR) OFFH-CDMA System

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Abstract— In this paper, a new method is proposed to analyze the cutoff rate of a variable-bit-rate (VBR) optical frequency hopping code division multiple access system (OFFH-CDMA) using fiber Bragg gratings and direct detection. This approach exploits the linear structure of passive optical CDMA systems and the nominal time required to accomplish the encoding-decoding operations in such systems. A system model is presented and analyzed based on a newly introduced bitoverlap procedure. An expression for the cutoff rate of a VBR OFFH-CDMA system is derived. It is shown that for a required quality of service (QoS) guarantee, the system's data rate can be increased beyond the nominal limit imposed by the physical constraint of the encoder-decoder set.

I. INTRODUCTION

DUE to the emerging demand for variable and hierarchical quality of service (QoS) optical fiber communication networks where data must be transferred with different transmission speeds, future optical services will likely integrate many different streams of traffic. For this reason, optical code division multiple access (CDMA) with variable-bit-rate (VBR) has received much attention lately [1]-[3].

It is important to emphasize the difference between passive optical CDMA and its electrical active counterpart in order to justify our work. In fact, in active CDMA systems there is a one-to-one correspondence between the transmitted symbol duration and the processing gain (PG) in the sense that changing the bit duration will eventually lead to a change in the users PG. On the other hand, this one-to-one relation does not exists in passive optical CDMA systems. For instance, decreasing the bit duration will not affect the symbol duration at the output of the optical encoder. Therefore, for a fixed PG, increasing the link transmission rate beyond a given value, known as the nominal rate, leads to bit overlap at the output of the encoder. Based on this concept, which is unique to passive optical CDMA, we determine the cutoff rate of an optical fast frequency hopping CDMA (OFFH-CDMA) system.

In [6], we have proposed a multirate OFFH-CDMA system using fiber Bragg grating and variable PG. The idea was to respect the total round trip time for light from a data bit to traverse the encoder. Our intention was to guarantee the one-to-one correspondence between the PG and the source transmission rate. The drawback of this system is the drastic decrease in the transmitted signal power especially for higher rate users for which the PG becomes very low. The solution to this problem is the use of power control [3].

On the other hand, *Zhang* in [1] and [2] introduced a novel technique that lead to the generation of a new family of Optical Orthogonal Codes (OOC) called the *Strict* OOC. He considered that a VBR could be achieved by varying the time delay between the transmitted data symbols. Although the *Strict* OOC ensures both auto- and cross-correlation constrains to be less or equal to one, the cutoff rate of the system is still limited by the physical constraints of the encoder-decoder set in a way the maximum transmission rate is achieved when the delay between two data symbols is equal to zero.

In this work, the general problem we consider is by how much we can increase the transmission rate beyond the nominal permitted one so as to optimize performance to meet the QoS requirement, given a fixed PG and number of active channels. The maximum achievable bit rate will be notified as the cutoff rate of the network. We will show that for an optimized family of codes, it is possible to increase the bit rate beyond the nominal rate without decreasing the PG as in [6] or allowing any time delay between the data symbols as proposed in [1].

Following the introduction, the paper is organized as follows. Section II presents the system model. In section III, we quantify the effective increase in the number of hits as a function of the transmission rate. An expression of the cutoff rate for a VBR OFFH-CDMA system is derived in Section IV. Section V contains some numerical results and discussions. Finally, the conclusion is presented in Section VI.

II. SYSTEM MODEL

Consider an OFFH-CDMA communication network that supports K users, which share the same optical medium in a star architecture [4]. The encoding and decoding are achieved passively using a sequence of fiber Bragg gratings. The gratings will spectrally and temporally slice an incoming broadband pulse into several components equally spaced at chip intervals $T_c = 2n_g L_c/c$ [4] as shown in Fig. 1. L_c represents the grating length assuming that the grating's temporal response is an ideal square wave function, c is the speed of light, and n_g is the group index. The chip duration, and the number of gratings will establish the nominal bit rate of the system, *i.e.* the round trip time of light, from a given transmitted bit, to be totally reflected from the encoder. This nominal bit duration in a structure of G gratings is given by $T_n = 2Gn_e L_c/c$, where G is the PG. The corresponding nominal rate is $R_n = 1/T_n$.



Fig. 1 OFFH-CDMA system.

Due to the linearity of the gratings hence the linearity of the encoder-decoder set, when the data rate increases beyond R_n , multi-bits will be coded during T_n and transmitted as revealed in Fig. 1. At a given receiver the decoder observes practically multicode, which are delayed according to the transmission rate of the source as shown in Fig. 2. When user k transmits using rate $R_s > R_n$, it introduces a bit overlap coefficient ε_s according to which the new rate is related to R_n through the following equation

Fig. 2 Observed codes at the desired receiver for a) the k^{th} channel and b) the desired signal.

Before continuing the analysis, let us impose some restrictions which help simplify the mathematical analysis and improve the clarity of the problem under consideration. We assume 1) a synchronous system and discrete rate variation, 2) a single class system, and 3) unit transmission power for all the users.

A. Signal Structure

We define $a_k(t, f)$ and $b_k(t)$ as the hopping pattern and the baseband signal, respectively, where t and f represent the time and frequency dimensions. From Fig. 2, the optical bit stream can be seen to be serial-to-parallel converted to v optical pulses. Since the desired user nominal time period is $T_n = GT_c$, we are interested only in modeling the k^{th} interfering channel during T_n . Because the bit b_X^k is delayed by $\tau_X = X(G - \varepsilon_s)T_c$, this suggests that the channel model, as seen by the desired receiver, can be represented as a tapped delay line with tap spacing of $\tau_{-1} = -(G - \varepsilon_s)T_c$ from left and $\tau_1 = (G - \varepsilon_s)T_c$ from right. The tap weight coefficients $b_X^k \in \{0,1\}$ depend on whether the transmitted bit is zero or one. The truncated tapped delay line model as seen by the desired receiver is shown in Fig. 3. Accordingly, the transmitted signal is given by

$$S_k(t,f) = \sum_{\nu} b_{\nu}^k a_k(t - \tau_{\nu}, f)$$
⁽²⁾

We define v and τ_v as the index of the tap coefficient and its associated time delay, respectively.



Fig. 3 Channel model.

Lemma 1: At the desired receiver end, and during the nominal time period T_n , the observed total number of taps in channel k, which transmits using rate R_s is

$$N_{k}(G,\varepsilon_{s}) = 2\left[\frac{\varepsilon_{s}}{G-\varepsilon_{s}}\right] + 1$$
(3)

where |x| is the smallest integer greater than x.

Proof: For a given transmission rate R_s , which corresponds to $0 \le \varepsilon_s \le G-1$ through (1), we can notice that in order for a transmitted bit b_x^k not to correlate with the desired user code during T_n , the following inequalities must be satisfied

1) Preceding bits from the right

$$X \ge \frac{G}{G - \varepsilon_s} \tag{4}$$

If we use the fact that we consider discrete chip overlap, the smallest integer that satisfies (4) is

$$X = \left[\frac{G}{G - \varepsilon_s}\right]$$

Thus, we can define the final bit $b_{X_r}^k$ that correlates with the desired decoder from the right as follows

$$X_r = \left[\frac{G}{G_s - \varepsilon_s}\right] - 1 = \left[\frac{\varepsilon_s}{G - \varepsilon_s}\right]$$

2) Upcoming bits from the left

The same analysis can be applied for the upcoming bits and the total number of the overlapped upcoming bits is

$$X_{l} = X_{r} = \left[\frac{\varepsilon_{s}}{G - \varepsilon_{s}}\right]$$

Therefore, the total number of observed transmitted codes is equal to X_r plus X_l in addition to the normal bit b_0^k , which proves (3).

The received signal at the input of the decoder is given by

$$y(t, f) = n(t) + \sum_{k=0}^{K-1} \sum_{\nu=-X_r}^{X_r} b_{\nu}^k a_k(t - \tau_{\nu}, f)$$

where n(t) is an additive white Gaussian noise (AWGN) with two-sided spectral density $N_0/2$.

B. Decoder's Output

Without loss of generality, we assume that the correlation-matched filter is matched to the zeroth signal. The output of the noncoherent matched filter correlator will be

$$Z_{0} = N + \int_{0}^{T_{n}} \sum_{k=0}^{K-1} S_{k} \left(t - \tau_{v}, f \right) a_{0} \left(t, f \right) dt$$

where *N* is a zero-mean AWGN with variance $\sigma_n^2 = N_0 T_n / 4$. The MAI I_k from user *k* that transmits data with rate R_s can be written as

$$I_{k} = \sum_{\nu=-X_{r}}^{-1} \int_{0}^{\tau_{\nu}} b_{\nu}^{k} h(a_{k}(t-\tau_{\nu}), a_{0}(t)) dt + \sum_{\nu=0}^{X_{r}} \int_{\tau_{\nu}}^{\tau_{n}} b_{\nu}^{k} h(a_{k}(t-\tau_{\nu}), a_{0}(t)) dt$$
(5)

 $\forall k \neq 0$. h(.) is the Hamming function defined in [6]. The sequences $a_k(t)$ and $a_0(t)$ are real numbers representing wavelengths used at time t for the k^{th} interferer and the desired user, respectively. Notice that $a_k(i) = a_k(i+T_s)$. In addition, we define a new performance parameter called the auto-interference, I_0 , caused by the desired user's signal as shown in Fig. 2b) and it is given by

$$I_{0} = \sum_{\nu=-X_{r}}^{T_{1}} \int_{0}^{\tau_{\nu}} b_{\nu}^{0} h(a_{0}(t-\tau_{\nu}), a_{0}(t)) dt + \sum_{\nu=1}^{X_{r}} \int_{\tau_{\nu}}^{T_{n}} b_{\nu}^{0} h(a_{0}(t-\tau_{\nu}), a_{0}(t)) dt$$
(6)

III. SIR PERFORMANCE EVALUATION

The MAI I_k , $\forall 0 \le k \le K-1$, is assumed to be an independent random variable. Hence, the variance of the decision variable Z_0 is

$$\operatorname{var}[Z_0] = \sum_{k=1}^{K-1} \sigma_{I_k}^2 + \sigma_{I_0}^2 + \sigma_n^2$$
(7)

 $\sigma_{I_k}^2$ and $\sigma_{I_0}^2$ represent the interference power caused by an active user *k* and the auto-interference power, respectively, and they are given by

$$\sigma_{I_k}^2 = E(I_k^2) - E^2(I_k) \quad \forall \ 0 \le k \le K$$
(8)

where E(.) is the expectation operator over all possible values of the overlapping bits b_X^k for $X \in \{-X_r, ..., X_r\}$ assuming that $\Pr(b_X^k = 1) = \Pr(b_X^k = 0) = 1/2$. Using the Frequency Shifted Version (FSV) system proposed in [5], $E(I_k)$ can be made equal to zero and the cross terms generated from squaring the summation in $E(I_k^2)$ become zeros, which enable us to write

$$E(I_{k}^{2}) = R_{k}(T_{n},\varepsilon_{s}) = \frac{1}{2} \left[\sum_{\nu=-X_{r}}^{-1} H_{k,0}^{2}(0,\tau_{\nu}) + \sum_{\nu=0}^{X_{r}} H_{k,0}^{2}(\tau_{\nu},T_{n}) \right]$$

$$E(I_{0}^{2}) = R_{0}(T_{n},\varepsilon_{s}) = \frac{1}{2} \left[\sum_{\nu=-X_{r}}^{-1} H_{0,0}^{2}(0,\tau_{\nu}) + \sum_{\nu=1}^{X_{r}} H_{0,0}^{2}(\tau_{\nu},T_{n}) \right]$$
(10)

where, $H_{k,0}(\tau_i, \tau_j)$ is the continuous-time partial-period Hamming-correlation function given by

$$H_{k,0}\left(\tau_{i},\tau_{j}\right) = \int_{\tau_{i}}^{\tau_{j}} h\left(a_{k}\left(t-\tau_{v}\right),a_{0}\left(t\right)\right)dt$$

Let $q_v = \tau_v / T_c$, we can write

$$H_{k,0}(0,\tau_{\nu}) = T_c H_{\nu}(0,q_{\nu}) = T_c \sum_{j=0}^{q_{\nu}-1} h(a_{j-q_{\nu}}^k,a_j^0)$$
(11)

$$H_{k,0}(\tau_{v},T_{n}) = T_{c}H_{v}(q_{v},G) = T_{c}\sum_{j=q_{v}}^{G-1}h(a_{j-q_{v}}^{k},a_{j}^{0}) \quad (12)$$

Using (11) and (12), $R_k(T_n, \varepsilon_s)$ and $R_0(T_n, \varepsilon_s)$ can be written as

$$R_{k}(T_{n},\varepsilon_{s}) = \frac{T_{c}^{2}}{2} \left[\sum_{\nu=-X_{r}}^{-1} H_{\nu}^{2}(0,q_{\nu}) + \sum_{\nu=0}^{X_{r}} H_{\nu}^{2}(q_{\nu},G) \right]$$
(13)

$$R_{0}(T_{n},\varepsilon_{s}) = \frac{T_{c}^{2}}{2} \left[\sum_{\nu=-X_{r}}^{-1} H_{\nu}^{2}(0,q_{\nu}) + \sum_{\nu=1}^{X_{r}} H_{\nu}^{2}(q_{\nu},G) \right]$$
(14)

If we define $R_k(G,\varepsilon_s) = R_k(T_n,\varepsilon_s)/(T_c^2/2)$, then we substitute into (8), the SIR experienced by any active user will be

$$\operatorname{SIR} = \frac{G^2}{\sum_{k=1}^{K-1} R_k \left(G, \varepsilon_s \right) + R_0 \left(G, \varepsilon_s \right) + \sigma_n^2}$$
(15)

A. Effective Increase in the Number of Hits

Proposition 1: In an OFFH-CDMA system that uses a family of codes, which respects the one-coincidence criterion and with non repeating frequencies in the same code [7], the expected value of the increase in the number of hits caused by any active interferer with (G, ε_s) on a desired user is given by

$$I_{H}^{k}(G,\varepsilon_{s}) = \frac{1}{F} \Big[(G+\varepsilon_{s}) X_{r} - (G-\varepsilon_{s}) X_{r}^{2} \Big]$$
(17)

and the effective increase of the number of hits due to the auto-interference is

$$I_{H}^{0}\left(G,\varepsilon_{s}\right)=0\tag{18}$$

where X_r is given throughout Lemma 1 and F is the total number of available frequencies.

Proof: The proof is omitted due to limited space.

In Fig. 4, we plot the position of hits between two Extended Hyperbolic Congruential (EHC) [7] codes with G = 40 and for two different transmission rates, a) $\varepsilon_s = 0$ and b) $\varepsilon_s = 35$.



Fig. 4 Hit positions between two EHC codes for a) $\varepsilon_s = 0$ and b) $\varepsilon_s = 35$.

B. Average SIR

If we consider the overlapping codes generated from an active user as independent virtual users, we can compute the average correlations given in (13) and (14) assuming one-coincidence sequences. The results are as follows

$$\begin{split} \bar{H}_{\nu}^{2}\left(0,q_{\nu}\right) &= E\left[\sum_{j=0}^{q_{\nu}-1}h^{2}\left(a_{j-q_{\nu}}^{k},a_{j}^{0}\right)\right.\\ &\left.+\sum_{j=0}^{q_{\nu}-1}\sum_{\substack{i=0\\i\neq j}}^{q_{\nu}-1}h\left(a_{i-q_{\nu}}^{k},a_{i}^{0}\right)h\left(a_{j-q_{\nu}}^{k},a_{j}^{0}\right)\right] \end{split}$$

Due to the fact that we are assuming one-coincidence sequences, the second term in the above expression is obviously equal to zero. In addition, $h^2\left(a_{j-q_i}^k, a_j^0\right)$ is a Bernoulli random variable with values taken from the set $\{1,0\}$ with probabilities $P(H_j^v)$ and $P(\overline{H}_j^v)$, respectively. Hence

$$\bar{H}_{\nu}^{2}(0,q_{\nu}) = \sum_{j=0}^{q_{\nu}-1} \Pr\left(a_{j-q_{\nu}}^{k} = a_{j}^{0}\right) = \sum_{j=0}^{q_{\nu}-1} \frac{1}{F} = \frac{G - \nu(G - \varepsilon_{s})}{F} \quad (19)$$

Following the same analysis, $H_{\nu}^{2}(q_{\nu},G)$ is given by

$$\bar{H}_{\nu}^{2}(q_{\nu},G) = \sum_{j=q_{\nu}}^{G-1} \frac{1}{F} = \frac{G-\nu(G-\varepsilon_{s})}{F}$$
(20)

If we substitute (19)-(20) into (13) and (14), we obtain

$$\overline{R}_{k}\left(G,\varepsilon_{s}\right) = \frac{1}{2F} \left[G + F \cdot I_{H}^{k}\left(G,\varepsilon_{s}\right)\right]$$
(21)

$$\overline{R}_0(G,\varepsilon_s) = 0 \tag{22}$$

where $I_{H}^{k}(G,\varepsilon_{s})$, is given in (17). Therefore, the average SIR for a system with (G,ε_{s}) will be

$$\operatorname{SIR} = \frac{G^2}{\frac{(K-1)}{2F}G + \frac{(K-1)}{2}I_H^k(G,\varepsilon_s) + \sigma_n^2}$$
(23)

IV. CUTOFF RATE

The main objective of this paper is to get a closed form solution for the cutoff rate of a VBR OFFH-CDMA system given K, G, and the QoS guarantee β . The method consists of solving for ε_s using (23). The problem is divided into two steps. The first one is when we consider that the required QoS allows the transmission rate to be $R_n \le R_s \le 2R_n$. In the second step we assume that the QoS requirement is small enough to allow $2R_n < R_s \le GR_n$. Thus, the critical value of ε_s that separates the two cases is $\varepsilon_{threshold} = G/2$ for which we can compute the threshold SIR as follows

$$\beta_{threshold} = \frac{G^2}{\left(K-1\right)\frac{G}{F} + \sigma_n^2}$$

Note that F must fulfill

$$F \ge (K-1)\beta/2G \tag{24}$$

otherwise no overlap is allowed.

1) $\beta \geq \beta_{threshold}$

In this case ε_s must satisfy $0 \le \varepsilon_s \le G/2$; therefore $X_r = 1$. In order to respect the QoS guarantee, the computed SIR for each user must respect SIR $\ge \beta$. Thus we can write the following inequality

$$\frac{G^2}{\frac{(K-1)G}{2F} + \frac{(K-1)\varepsilon_s}{F} + \sigma_n^2} \ge \beta$$

By taking the equality and solving for ε_s , we obtain the cutoff rate as follows

$$\varepsilon_{cutoff} = \frac{FG^2}{(K-1)\beta} - \frac{G}{2} - \frac{F\sigma_n^2}{(K-1)} \quad \forall \beta \ge \beta_{threshold} \quad (25)$$

2)
$$\beta < \beta_{threshold}$$

For this situation ε_s must fulfill $G/2 < \varepsilon_s < G$. The problem seems to be more complicated due the integer problem represented by X_r .

Proposition 2: For any $G/2 < \varepsilon_s < G$, as ε_s tends toward G, the SIR expression given in (23) converges to

$$\operatorname{SIR} = \frac{G^2}{\left(K-1\right)\frac{G}{2F} + \left(K-1\right)\frac{G\varepsilon_s}{2\left(G-\varepsilon_s\right)F} + \sigma_n^2}$$
(26)

Proof: If $G/2 < \varepsilon_s < G$, the integer value X_r is lower bounded by $X_{r_{min}} = \varepsilon_s/G - \varepsilon_s$ and upper bounded by $X_{r_{max}} = G/G - \varepsilon_s$. Using some simple algebra, it can be easily shown that

$$I_{H}^{k}\left(X_{r}=X_{r_{max}}\right)=I_{H}^{k}\left(X_{r}=X_{r_{min}}\right)=\frac{G\varepsilon_{s}}{\left(G-\varepsilon_{s}\right)F}$$

Knowing that I_{H}^{k} is a second order equation in X_{r} , the maximum of I_{H}^{k} occurs at the extreme $X_{r_extreme} = (G + \varepsilon_{s})/2(G - \varepsilon_{s})$ for which we can write

$$I_{H_{\rm max}}^{k} = \frac{1}{4F} \frac{\left(G + \varepsilon_{s}\right)^{2}}{\left(G - \varepsilon_{s}\right)}$$

Thus, the exact value of X_r is either bounded by $X_{r_{min}} \leq X_r \leq X_{r_{extreme}}$ or $X_{r_{extreme}} \leq X_r \leq X_{r_{max}}$. Therefore, in the two cases, I_H^k is bounded by

$$\frac{G\varepsilon_{s}}{\left(G-\varepsilon_{s}\right)F} \leq I_{H}^{k}\left(X_{r}\right) \leq \frac{1}{4F} \frac{\left(G+\varepsilon_{s}\right)^{2}}{\left(G-\varepsilon_{s}\right)}$$

We define the relative measure of the tightness between the upper and lower bound of I_H^k as

$$\Delta(G,\varepsilon_s) = \frac{I_{H_{-}\max}^k - I_{H_{-}\min}^k}{I_{H_{-}\min}^k} = \frac{(G-\varepsilon_s)^2}{4G\varepsilon_s}$$

We can notice clearly that $\lim_{\varepsilon_s \to G} \Delta(G, \varepsilon_s) = 0$, which makes the two bounds converges asymptotically to the same value

$$I_{H}^{k} = \frac{G\varepsilon_{s}}{\left(G - \varepsilon_{s}\right)F}$$

Using this result in (23), we obtain (26).

In Fig. 5, we plot I_H^k versus ε_s . For $\varepsilon_s > G/2$, and as ε_s increases, the bounds are very tight. In addition, it is clear that as *F* increases, I_H^k decreases; therefore, enabling higher transmission rate for the network.



Fig. 5 Effective increase in the number of hits versus the overlapping coefficient and for different value of *F*.

Hence, using the lower bound of X_r and assuming that $\sigma_n^2 = 0$, we obtain an upper bound of ε_s as follows

$$\frac{G^2}{(K-1)\frac{G}{2F} + (K-1)\frac{G\varepsilon_s}{2(G-\varepsilon_s)F}} \ge \beta$$

Therefore, the upper bound of ε_s is

$$\varepsilon_{upper} = \frac{2FG - (K-1)\beta}{2F}$$
(27)

On the other hand, a lower bound of the cutoff rate can be derived by using $X_{r.extreme}$. Hence we can write

$$\frac{G^2}{(K-1)\frac{G}{2F} + (K-1)\frac{(G+\varepsilon_s)^2}{8(G-\varepsilon_s)F}} \ge \beta$$

 ε_{lower} is obtained by solving the following second order equation in ε_{e}

$$(K-1)\beta\varepsilon_s^2 + (8FG^2 - 2(K-1)G\beta)\varepsilon_s + (2(K-1)G^2\beta - 8FG^3) = 0$$
(28)

Note that the condition in (24) insures the existence of a valid solution for (28), which is given by

$$\varepsilon_{lower} = \frac{-4FG^2 + 2G\sqrt{4F^2G^2 - (K-1)^2\beta^2}}{(K-1)\beta} + G \quad (29)$$

Thus, the cutoff rate of the system is given by

$$\varepsilon_{lower} \le \varepsilon_{cutoff} \le \varepsilon_{upper} \quad \forall \ \beta < \beta_{threshold} \tag{30}$$

It is important to note that (30) represents the bottleneck of the transmission rate for a single class OFFH-CDMA system. Asymptotically, As *F* increases, it is easily seen that $\lim_{E \to \infty} \varepsilon_{cutoff} = G$. Hence, the system allows full overlap.

V. NUMERICAL RESULTS AND DISCUSSIONS

In comparison to previous works [1]-[6], an important contribution of this paper lies in exploiting the linear structure and the physical constraints of passive optical CDMA.

It can be seen from Fig. 6 the importance of the QoS requirement in determining ε_{cutoff} of the system. Observe that for $\beta > 100$ and assuming F = 40 and K = 30, the system does not allow any increase in the transmission rate beyond R_n . On the other hand, when $\beta < 60$, the system may allow more than $2R_n$. Note also the importance of F in determining ε_{cutoff} . ε_{cutoff} increases asymptotically as F becomes very high, which is in total agreement with the analytical results discussed in Section IV.



Fig. 6 Cutoff overlapping coefficient versus the QoS requirements, and for different value of F.



Fig. 7 SIR versus *F* and ε_s .

In Fig. 7 we plot the SIR as a function of F and ε_s , for K = 30. Notice that as F increases beyond G = 40, the

system's SIR increases also due to the decrease in the probability of hit. If we assume that $\beta = 20 \text{ dB}$, we can determine ε_{cutoff} of the system function of *F* as shown by the intersection line between the plane and the SIR surface. Observe the asymptotic behavior of ε_s as *F* becomes very high. It is clear that if we keep on increasing *F*, ε_{cutoff} will eventually reach *G*.

VI. CONCLUSION

In this work, the main idea from our derivations is to find and analyze the cutoff rate for a VBR OFFH-CDMA system. By taking into account the virtual users induced by an interfering channel and the desired source, the system's SIR was derived. Based on this SIR, we have been able to obtain a closed form solution for the cutoff rate of a single class system. Simulation and analytical results showed that for given QoS requirements, number of available frequencies, and number of active channels, it is possible to increase the transmission rate well beyond the nominal rate imposed by the physical dimensions of the encoder/decoder pairs.

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