Optical CDMA with Interference Cancellation

Hossam M. H. Shalaby, Member, IEEE

Department of Electrical Engineering, University of Alexandria, Alexandria 21544, Egypt.

Abstract—A multiple-user interference reduction technique is proposed for optical code-division multiple-access systems. Data symbols from each user are encoded using pulse-position modulation scheme before multiplexing. Modified prime sequences are adopted as the signature codes in the multiplexing process. An interesting property of this code is the uniformity of the cross-correlation among its sequences. This property is main key in constructing the multiple-access interference canceller. In addition to its simplicity, this canceller offers a great improvement in the error probability as compared to the system without cancellation.

I. Introduction

There is an increasing interest in utilizing code-division multiple-access (CDMA) techniques in fiber-optic local area networks [1–15]. This is because of the wide bandwidth offered by the optical components. Both synchronous [1–6] and asynchronous [7–15] techniques have been studied in literature. Synchronous optical CDMA has some advantages over asynchronous CDMA [2]. Namely, both the possible number of subscribers and the number of simultaneous users (that can be accommodated for a given probability of error) are greater in the case of synchronous CDMA. Synchronization subsystems, however, are mandatory for the synchronous system. Thus in high data rate applications where synchronization can be achieved easily, e.g., LANs, synchronous optical CDMA stands as an attractive candidate.

Degradation in the performance of optical CDMA systems, even for ideal ones, is essentially due to the multiple access interference. This type of interference results from the incomplete orthogonality of the used signature codes. If the receiver is able to extract an estimate for this interference, it could cancel or reduce its effect leading to an improvement in the performance.

The process of interference cancellation or reduction is not an easy task in general because it involves estimations of data symbols for each subscriber. This obviously increases the complexity of the receiver as the number of subscribers increases. Modified prime codes has an interesting property where the entire set of codes can be divided into groups. The users within one group are completely orthogonal whereas any two users from two different groups are not orthogonal. In this paper we utilize this property in developing a simple interference canceller. In fact this canceller provides an estimation about the interference with the aid the users sharing the same group only.

To further improve the performance of the CDMA system the data symbols are encoded using pulse-position modulation (PPM) scheme before multiplexing. We have studied extensively the performance of optical PPM-CDMA system without cancellation in [3]. Optical orthogonal codes [7] have been employed in this study. In [6] we have developed three types of interference cancellation for the binary on-off keying CDMA systems. Ideal photodetectors are assumed in the analysis of [6].

In this paper we suggest simple direct-detection optical PPM-CDMA systems with interference cancellation. We employ the modified prime sequences as our signature codes. The Poisson effects of the photodetection process is considered in our analysis. Comparisons between systems with an ewithout interference cancellation are examined as well.

The rest of the paper is organized as follows. A system model for optical PPM-CDMA without interference cancellation is described in Section II along with a derivation for a lower bound on its bit error rate. Section III is devoted for the description of the proposed interference canceller. Upper bound on the probability of error for this canceller is developed in this section as well. Numerical results and performance comparisons are demonstrated in Section IV. Finally our conclusions and findings are given in Section V.

II. OPTICAL PPM-CDMA WITHOUT INTERFERENCE CANCELLATION

We start be recalling some of the properties of the modified prime sequences. The interested reader may refer to [2] for further details. Let a prime number p be given. There are p^2 modified prime sequences that can be generated. Each code sequence has a weight equals to p and a length p^2 . The codes are divided into p groups, each group consists of p different codes. The cross-correlation (C_{mn}) between code m and code n is given by

$$C_{mn} = \begin{cases} p ; & \text{if } m = n, \\ 0 ; & \text{if } m \text{ and } n \text{ share the same group} \\ & \text{and } m \neq n, \\ 1 ; & \text{if } m \text{ and } n \text{ are from different groups.} \end{cases}$$
 (1)

Since we have at most p^2 sequences, the total number of subscribers is thus equal to p^2 . Out of this number we assume that there are N active (simultaneous) users and the remaining p^2-N users are assumed idle. Each active user is assigned a code sequence randomly with a uniform distribution. This sequence is called the address or signature of the

user. We define a random variable γ_n , $n \in \{1, 2, ..., p^2\}$, as follows

$$\gamma_n = \begin{cases} 1 ; & \text{if user } n \text{ is active,} \\ 0 ; & \text{if user } n \text{ is idle.} \end{cases}$$

Thus

$$\sum_{n=1}^{p^2} \gamma_n = N .$$

We assume for simplicity that user one is the desired user $(\gamma_1 = 1)$. Let the random variable T represent the number of active users in the first group:

$$T \stackrel{\mathrm{def}}{=} \sum_{n=1}^{p} \gamma_n \ . \tag{2}$$

It is easy to check that the probability distribution of this random variable, given that user 1 is active, can be written as

$$P_T(t) = \frac{\binom{p^2 - p}{N - t}\binom{p - 1}{t - 1}}{\binom{p^2 - 1}{N - 1}}, \qquad t \in \{t_{\min}, t_{\min} + 1, \dots, t_{\max}\},$$
(3)

where

$$t_{\min} \stackrel{\text{def}}{=} \max\{N+p-p^2,1\}$$
 and $t_{\max} \stackrel{\text{def}}{=} \min\{N,p\}$.

In M-ary PPM-CDMA signaling format [3], a time frame of duration T is divided into M disjoint slots each having a width $\tau = T/M$. Symbol $i \in \{0, 1, ..., M-1\}$ is represented by signaling a single laser pulse of width $T_c = \tau/p^2$ at the leading edge of slot number i. This pulse is further spread into p laser pulses, p is a prime number. The spreading process could be performed with the aid of a splitter, a tapped delay line, and a combiner. The width of each of the resulting pulses is also T_c . The relative positions of these pulses are determined according to the corresponding signature code. Thus the underlined slot contains a sequence of optical pulses representing the code and all other slots contain nothing.

Let \mathbf{D}_n be a vector of length M representing the data symbol for user n. If user n wishes to send symbol $i \in \{0,1,\ldots,M-1\}$, then each entry in \mathbf{D}_n will be equal to zero except the ith entry will be equal to one. That is $D_{n,i}=1$ and $D_{n,j}=0$ for every $j\neq i$. As explained in [3], receiver one correlates the compound received sequences of laser pulses, in each slot, with the corresponding signature code. This results in the collection of M photon counts. We denote by $Y_{1,i}$ the photon count collected over slot i, for every $i\in\{0,1,\ldots,M-1\}$. Moreover, we denote by the vector \mathbf{Y}_1 the collection of the random variables $(Y_{1,0},Y_{1,1},\ldots,Y_{1,M-1})$ over all slots. Thus, $\{Y_{1,i}\}_{i=0}^{M-1}$ are independent Poisson random variables and \mathbf{Y}_1 is a Poisson random vector. We denote the mean of this random vector by the vector \mathbf{Z}_1 . Whence

$$\mathbf{Z}_1 = Qp \; \mathbf{D}_1 + Q \sum_{n=2}^{p^2} C_{1n} \mathbf{D}_n \gamma_n \; ,$$

where Q denotes the average received photon count per pulse. The last term in the above equation is due to the interference and will be represented by the random vector κ . Thus according to (1)

$$\mathbf{Z}_1 = Qp \; \mathbf{D}_1 + Q\kappa \; ,$$

where

$$\kappa \stackrel{\text{def}}{=} \sum_{n=p+1}^{p^2} \mathbf{D}_n \gamma_n \ .$$

In the subsequent analysis we assume equally likely data symbols. Thus given T = t, it is easy to check that κ is a multinomial random vector with probability

$$P_{\kappa|T}(l_0,\ldots,l_{M-1}|t) = \frac{1}{M^{N-t}} \cdot \frac{(N-t)!}{l_0! l_1! \ldots l_{M-1}!} , \quad (4)$$

where $\sum_{i=0}^{M-1} l_i = N - t$.

The Decision Rule

We employ the following decision rule: Symbol i is declared to be the correct one if $Y_{1,i} > Y_{1,j}$ for every $j \neq i$. The probability of bit error can thus be lower bounded as follows:

$$P_b = \frac{M}{2(M-1)} \sum_{t=t_{min}}^{t_{max}} P_E^t P_T(t) ,$$

where

$$\begin{split} P_E^t &= \sum_{i=0}^{M-1} \Pr\{Y_{1,j} \geq Y_{1,i}, \text{ some } j \neq i | T = t, D_{1,i} = 1\} \\ &\quad \times \Pr\{D_{1,i} = 1\} \\ &= \Pr\{Y_{1,j} \geq Y_{1,0}, \text{ some } j \neq 0 | T = t, D_{1,0} = 1\} \\ &\geq \Pr\{Y_{1,1} \geq Y_{1,0} | T = t, D_{1,0} = 1\} \\ &= \sum_{\mathbf{l}} P_{\kappa|T}(\mathbf{l}|t) \Pr\{Y_{1,1} \geq Y_{1,0} | T = t, \kappa = \mathbf{l}, D_{1,0} = 1\} \;, \end{split}$$

where I denotes the vector $(l_0, l_1, \ldots, l_{M-1})$ and

$$\Pr\{Y_{1,1} \ge Y_{1,0} | T = t, \kappa = 1, D_{1,0} = 1\} = \sum_{y_1=0}^{\infty} e^{-Ql_1} \frac{(Ql_1)^{y_1}}{y_1!} \sum_{y_1=0}^{y_1} e^{-Q(p+l_0)} \frac{(Q(p+l_0))^{y_0}}{y_0!}.$$

It is obvious that P_E^t decreases as Q increases. Taking the limit as $Q \to \infty$, we obtain the following lower bound:

$$P_{E}^{t} \geq \sum_{l_{1}=p+1}^{N-t} {N-t \choose l_{1}} \frac{1}{M^{l_{1}}} \left(1 - \frac{1}{M}\right)^{N-t-l_{1}} \times \sum_{l_{0}=0}^{\min\{l_{1}-p-1,N-t-l_{1}\}} {N-t-l_{1} \choose l_{0}} \frac{1}{(M-1)^{l_{0}}} \times \left(1 - \frac{1}{M-1}\right)^{N-t-l_{0}-l_{1}} + 0.5 \sum_{l_{1}=p}^{\frac{N-t+p}{2}} {N-t \choose l_{1}} \frac{1}{M^{l_{1}}} \left(1 - \frac{1}{M}\right)^{N-t-l_{1}} \times {N-t-l_{1} \choose l_{1}-p} \frac{1}{(M-1)^{l_{1}-p}} \times \left(1 - \frac{1}{M-1}\right)^{N-t-2l_{1}+p}.$$

$$(5)$$

III. INTERFERENCE REDUCTION IN PPM-CDMA

To understand the basic idea of our interference canceller we notice that the mean vectors $\{\mathbf{Z}_n\}_{n=2}^p$ for the photon count vectors $\{\mathbf{Y}_n\}_{n=2}^p$ collected by the users sharing the same group with user 1 are given by

$$\mathbf{Z}_n = Qp \; \mathbf{D}_n \gamma_n + Q\kappa \; .$$

That is the average photon counts due to the interference κ are the same for all users in one group. This suggests constructing the vector $\tilde{\mathbf{Y}}_1$ as follows:

$$\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \frac{1}{p-1} \sum_{n=2}^p \mathbf{Y}_n \ . \tag{6}$$

The block diagram of this canceller is shown in Fig. 1, where in this case the decision rule for user 1 is processed aided with the entries of $\tilde{\mathbf{Y}}_1$ rather than \mathbf{Y}_1 . Of course we assume that all the signature codes of the same-group users are known to each other.

We define the random vector \mathbf{X} as

$$\mathbf{X} \stackrel{\mathrm{def}}{=} \sum_{n=2}^{p} \mathbf{D}_{n} \gamma_{n} .$$

Given T = t, it is easy to check that **X** is again a multinomial random vector with probability

$$P_{\mathbf{X}|T}(x_0, \dots, x_{M-1}|t) = \frac{1}{M^{t-1}} \cdot \frac{(t-1)!}{x_0! x_1! \dots x_{M-1}!} , \quad (7)$$

where $\sum_{i=0}^{M-1} x_i = t-1$. Moreover, define the random vector \mathbf{R}_1 as

$$\mathbf{R}_1 = \sum_{n=2}^p \mathbf{Y}_n \ .$$

Given $\kappa = 1$ and $\mathbf{X} = \mathbf{x}$, it is obvious that \mathbf{R}_1 is an independent Poisson random vector with mean vector

$$\mathbf{S}_1 = \sum_{n=2}^p \mathbf{Z}_n = Qp\mathbf{X} + Q(p-1)\kappa \ .$$

 $\tilde{\mathbf{Y}}_1$ in (6) can now be written as

$$\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \frac{\mathbf{R}_1}{p-1} \ .$$

The Decision Rule

Similar to the case without cancellation we adopt the following decision rule: Symbol i is declared to be the correct one if $\tilde{Y}_{1,i} > \tilde{Y}_{1,j}$ for every $j \neq i$. We derive an upper bound on the bit error probability as follows.

$$P_b = \frac{M}{2(M-1)} \sum_{t=t_{\min}}^{t_{\max}} P_E^t P_T(t) , \qquad (8)$$

where

$$\begin{split} P_E^t &= \sum_{i=0}^{M-1} \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,i}, \text{ some } j \neq i | T = t, D_{1,i} = 1\} \\ &\times \Pr\{D_{1,i} = 1\} \\ &= \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,0}, \text{ some } j \neq 0 | T = t, D_{1,0} = 1\} \\ &= \sum_{x_0} P_{X_0|T}(x_0|t) \Pr\{\tilde{Y}_{1,j} \geq \tilde{Y}_{1,0}, \\ &\text{ some } j \neq 0 | T = t, X_0 = x_0, D_{1,0} = 1\} \\ &\leq P_{X_0|T}(p-1|t) + (M-1) \sum_{x_0 \neq p-1} P_{X_0|T}(x_0|t) \\ &\times \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T = t, X_0 = x_0, D_{1,0} = 1\} \\ &\leq P_{X_0|T}(p-1|t) + (M-1) \sum_{x_0 \neq p-1} P_{\kappa|T}(1|t) P_{\mathbf{X}|T}(\mathbf{x}|t) \\ &\times \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T = t, \kappa = 1, \mathbf{X} = \mathbf{x}, D_{1,0} = 1\} \\ &\leq P_{X_0|T}(p-1|t) + \sum_{x_0 \neq p-1} P_{\kappa|T}(1|t) P_{\mathbf{X}|T}(\mathbf{x}|t) \phi(t,1,\mathbf{x}), \end{split}$$

where

$$\begin{split} \phi(t,\mathbf{l},\mathbf{x}) &\stackrel{\mathrm{def}}{=} \\ (M-1) & \Pr\{\tilde{Y}_{1,1} \geq \tilde{Y}_{1,0} | T=t, \kappa=\mathbf{l}, \mathbf{X}=\mathbf{x}, D_{1,0}=1\} \ , \end{split}$$

I denotes the vector $(l_0, l_1, \ldots, l_{M-1})$, and \mathbf{x} denotes the vector $(x_0, x_1, \ldots, x_{M-1})$. Of course $P_{X_0|T}(p-1|t) = 0$ if $t \neq p$. $\phi(t, \mathbf{l}, \mathbf{x})$ can further be upper bounded as

$$\phi(t, \mathbf{l}, \mathbf{x}) = (M - 1) \Pr\{Y_{1,1} - \frac{R_{1,1}}{p - 1} \ge Y_{1,0} - \frac{R_{1,0}}{p - 1} | T = t, \kappa = \mathbf{l}, \mathbf{X} = \mathbf{x}, D_{1,0} = 1\}$$

$$\le (M - 1) E\left\{z^{\left(Y_{1,1} - \frac{R_{1,1}}{p - 1} - Y_{1,0} + \frac{R_{1,0}}{p - 1}\right)} | T = t, \kappa = \mathbf{l}, \mathbf{X} = \mathbf{x}, D_{1,0} = 1\right\},$$

for every z > 1. We remark that the last inequality can be justified by using Chernoff bound and E denotes the expected value. Performing the expectation yields

$$\log \phi(t, \mathbf{l}, \mathbf{x}) \leq \log(M - 1) - Q l_1 (1 - z) - Q(p + l_0) (1 - z^{-1}) - Q((p - 1)l_0 + p x_0)) (1 - z^{\frac{1}{p-1}}) - Q((p - 1)l_1 + p x_1)) (1 - z^{-\frac{1}{p-1}}) .$$
(9)

Setting $z = 1 + \delta$, where $\delta > 0$, we obtain

$$1 - z^{-1} \ge \delta - \delta^2$$
, $1 - z^{\frac{1}{p-1}} \ge -\frac{\delta}{p-1}$,

and

$$1 - z^{-\frac{1}{p-1}} \ge \frac{\delta}{p-1} - \frac{p \, \delta^2}{2(p-1)^2} \, .$$

Substituting in (9) and searching for the tightest (optimum) δ yield

$$\phi(t, \mathbf{l}, \mathbf{x}) \le (M - 1)e^{-Q\delta \mathcal{E}}$$
,

where

$$\mathcal{E} = 0.5 \left[p + \frac{p}{p-1} (x_1 - x_0) \right]$$

and

$$\delta = \frac{p + \frac{p}{p-1}(x_1 - x_0)}{2\left[p + l_0 + \frac{p}{2(p-1)}(l_1 + \frac{p}{p-1}x_1)\right]}.$$

From the above discussion the upper estimate on P_E^t reduces to

$$P_{E}^{t} \leq P_{X_{0}|T}(x_{0}|t) + (M-1) \times \sum_{\substack{l_{0},l_{1},x_{0},x_{1}:\\x_{0}\neq p-1}} P_{\kappa_{0},\kappa_{1}|T}(l_{0},l_{1}|t) P_{X_{0},X_{1}|T}(x_{0},x_{1}|t) e^{-Q\delta\mathcal{E}} .$$

$$(10)$$

We notice that δ is always positive as long as $x_0 \neq p-1$. Thus as $Q \to \infty$

$$P_E^t \rightarrow \left\{ \begin{array}{ll} 0 \ ; & \text{if} \ t < p, \\ P_{X_0|T}(p-1|p) = \frac{1}{M^{p-1}} \ ; & \text{if} \ t = p. \end{array} \right.$$

Whence

$$\lim_{Q \to \infty} P_b = \frac{\binom{p^2 - p}{N - p}}{\binom{p^2 - 1}{N - 1}} \cdot \frac{1}{2(M - 1)M^{p - 2}} \ .$$

IV. NUMERICAL RESULTS

Performance comparisons between optical PPM-CDMA systems with and without cancellation have been evaluated numerically with the aid of previous sections. For the system with cancellation, upper bounds on the bit error rate have been evaluated. For the system without cancellation, however, lower bounds have been calculated. Moreover, Q is taken equal to ∞ in the case of no cancellation. Demonstrations about these comparisons are given in Figs. 2-5. In Figs. 2 and 3 the bit error rate is plotted versus the average photons per nat, μ . This is related to Q by $\mu = Qp/\log M$. It is obvious from these two figures that the error probability improves remarkably with cancellation especially for large values of μ . In Fig. 4 we plot the bit error rate versus N. It is clear that without cancellation the system becomes not reliable as N increases. With cancellation, however, reliability is preserved as long as μ and/or M are large enough. Fig. 5 compares the performance for the case of full load $(N = p^2)$. This is plotted versus the prime number p. Again significant improvement appears with cancellation, making the system even reliable for the full capacity case. Conversely, with no cancellation one can not reach the full load and retain reliable transmission. Furthermore, with cancellation the performance improves as p increases so that an arbitrary small error probability can be achieved with p large enough.

V. Conclusion

An interference cancellation technique has been proposed for synchronous optical PPM-CDMA communication systems. The technique utilizes the grouping property of the modified prime sequence codes. Since the signature codes of the same-group users are orthogonal, the desired subscriber collects photodetector outputs from those users and subtracts them from a scaled version of the received signal. Bit error rates, for systems with and without cancellation, have been derived and compared. Our results demonstrate significant improvement in performance when using cancellation. Namely, we have shown that a prime sequence code always exists so that all the subscribers are able to communicate simultaneously with arbitrary small error probability.

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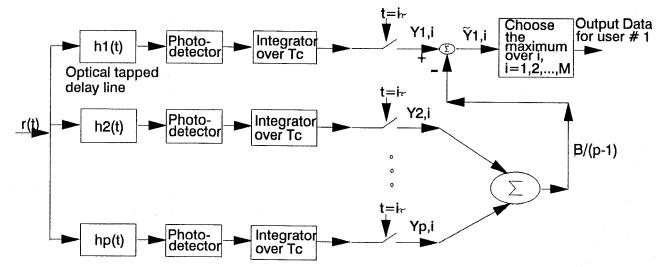


Fig. 1. Direct-detection optical PPM-CDMA system model with interference cancellation.

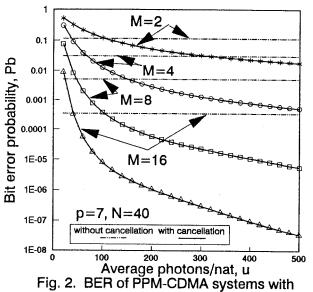


Fig. 2. BER of PPM-CDMA systems with and without cancellation for p=7 and N=40.

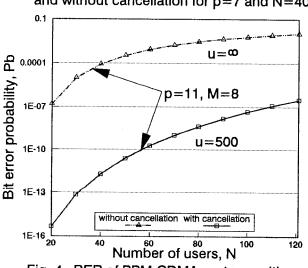


Fig. 4. BER of PPM-CDMA systems with and without cancellation for p=11 and M=8.

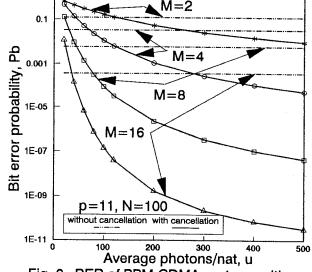


Fig. 3. BER of PPM-CDMA systems with and without cancellation for p=11 and N=100.

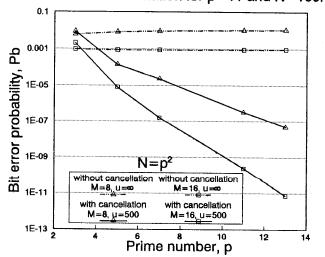


Fig. 5. BER of full capacity (N=p*p) PPM-CDMA systems with and without cancellation.