A Proposal of an Optical CDMA Random Access Protocol

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Abstract—**A random-access protocol for an optical direct-detection code-division-multiple access packet network is proposed. A mathematical description of the protocol is presented using a detailed state diagram. The steady-state system throughput is derived based on the equilibrium point analysis technique. The effect of several design parameters on the above performance measure has been examined with the aid of a set of numerical examples. In our numerical calculations chip-level models have been adopted at the receiver side.**

I. Introduction

Due to the vast bandwidth offered by the optical fibers and the extra-high optical signal processing speed bestowed by the optical components, optical code-division multiple-access (CDMA) techniques have been given an intensifying interest in the last decade [1]–[4]. Consequently, a large number of simultaneous users can be accommodated in local- and metropolitan-area networks (LANs and MANs) that employ optical CDMA techniques.

In [4] we have proposed two different protocols for slotted optical CDMA packet networks that needed pre-transmission coordination. A variant of one of these two protocols that does not need pretransmission coordination has also been described in [4]. In our analysis of the performance of these protocols, we have over-simplified the system in order to have some insight on the problem. Indeed we neither answered the following questions, nor did we take their effect on the performance evaluation in [4]:

1. How the transmitter deals with multi-packet messages.
2. How the transmitter responds to an arrived message.
3. How the transmitter manages lost packets or packets received with errors.
4. How the receiver responds to a request for communication.
5. How the propagation delay and the transmitter/receiver tuning time affect the network performance.

In this paper we aim at answering the above questions and exploring their effect on the overall performance. To answer the above questions we first describe the detailed architecture of an optical CDMA network. Next we propose a new optical CDMA protocol which takes care of the above questions. To explore the effect of these issues on the network performance, we face a prohibitively large number of states, which makes the problem analytically intractable. Fortunately, the equilibrium point analysis (EPA) technique significantly simplifies the problem and makes it more tractable. In this technique, the system is always assumed to be operating at an equilibrium point [5]. That is at any time slot, the expected number of users entering any state is always equal to that departing from the state.

A. System Architecture

The basic architecture of our optical CDMA network is composed of a set of N nodes or users, an optical star network, and a set of optical-orthogonal codes (OOCs) $C = \{a_1, a_2, a_3, \ldots, a_{|C|}\}$ with cardinality $|C|$. The cardinality $|C|$ depends on the code length L, the code weight w, and both the out-of-phase autocorrelation and cross-correlation constraints λ_a, λ_c, respectively. Traditionally $\lambda_a = \lambda_c = 1$, which gives [1]:

$$|C| = \left\lfloor \frac{L-1}{w(w-1)} \right\rfloor , \qquad (1)$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x. Since under normal situations the network users send their data in a burst mode, i.e., they are not all active at the same time, we allow the total number of users to exceed the number of available codes, $N > |C|$. Codes are assigned to all users a priori. That is when a user subscribes to the network, it is given a code (possibly used) randomly from C. Furthermore, the code is randomly cyclic shifted around itself once assigned.

Each user is assigned a fixed CDMA encoder and a tunable CDMA decoder (FT-TR). The transmitter is able to generate an optical On-Off keying CDMA (OOK-CDMA) signal (according to its signature code) that represents its data. The tunable receiver is able to tune to any code of all other transmitters. The structure of OOK-CDMA tunable receivers has been thoroughly studied in [2]. Indeed, Zhang [2] has used a set of electro-optic switches in his receiver implementation. He has shown that for a code with length L and weight w, only $(w-1)\{\lceil \log_2 L \rceil + 1\}$ switches are required, where $\lceil x \rceil$ denotes the smallest integer not less than x. In addition he has shown that the reconfiguration time of his tunable receiver is very fast and can be as low as 20 ns.

B. Optical CDMA Protocol

We impose the following assumptions in our model for optical CDMA protocol:

- Time is slotted with slot size T_s.
- Messages arrive to a station (or node) with probability A, also called *users activity*.
- A message is composed of a fixed number of packets, ℓ.
- A packet is composed of K bits. One packet should fit in a time slot. Thus the packet time slot $T_s = KLT_c$, where L is the code length and T_c is the chip time duration.
- Each node has a single buffer to store a message. Any arrival to a nonempty buffer is disregarded.
- A nonempty buffer is freed once the stored message is transmitted *successfully*.
- A node that has a message to transmit, first transmit a connection request to the destination station.
- The receiver of an idle station scans across all codes for connection requests in a round-robin (or polling) manner.
- At any time slot, if there is a connection request at a certain code, the station proceeds to send an acknowledgment.
- If there is no requests and there is a message arrival, the station tunes its receiver to the destination code and transmits a connection request on its signature code.
- Once an acknowledgment is received, the station proceeds to transmit its message packet by packet (each packet occupies a time slot).
- Every time a packet is received successfully at the destination, the receiver transmits an acknowledgment.
- The transmitting station follows a go-back n protocol for its transmission with n depending on the propagation time.
- A two way propagation time is assumed to be equal to t time slots.
- Transmission times for requests and acknowledgment data are neglected.

C. Chip-Level Receiver

Because of its superiority over the correlation model, we assume that the decision rule of any receiver depends on the chip-level model [3],[4]. That is, a data bit 1 is decided if the number of pulses in each weighted chip $Z_i, i \in \mathcal{X} = \{1, 2, \ldots, w\}$, of a code is nonzero [3]. Otherwise a data bit 0 is declared. Assuming that during a given time slot there are $r \in \{1, 2, \ldots, N\}$ active users, we define $\kappa \in \{0, 1, \ldots, r-1\}$ and $m \in \{0, 1, \ldots, r-1-\kappa\}$ as the number of users that interfere with the desired user at exactly 1 chip and w chips, respectively. Furthermore, let $\kappa_i, i \in \mathcal{X}$ denotes the number of users (out of κ users) that interfere with weighted chip i. Of course $\kappa = \sum_{i=1}^{w} \kappa_i$. We

have shown in [4] that the packet success probability for $r \in \{1, 2, \ldots, N\}$ active users is given by

$$
\begin{aligned}
P_s(r) = & \sum_{\kappa=0}^{r-1} \sum_{m=0}^{r-1-\kappa} \frac{(r-1)!}{\kappa! m! (r-1-m-\kappa)!} \\
& \cdot p_1^\kappa p_w^m (1 - p_1 - p_w)^{r-1-\kappa-m} \\
& \cdot \sum_{\substack{\kappa_1, \kappa_2, \ldots, \kappa_w: \\ \kappa_1 + \ldots + \kappa_w = \kappa}} \frac{\kappa!}{\kappa_1! \ldots \kappa_w!} \cdot \left(\frac{1}{w}\right)^\kappa \\
& \cdot \left[\frac{1}{2} + \frac{1}{2^{m+1}} \left(\sum_{i=1}^{w} \frac{1}{2^{\kappa_i}} - \sum_{i=1}^{w-1} \sum_{j=i+1}^{w} \frac{1}{2^{\kappa_i + \kappa_j}} \right. \right. \\
& \left. \left. + \ldots + (-1)^{w-1} \frac{1}{2^\kappa} \right) \right]^K,
\end{aligned}
\tag{2}
$$

where p_1 and p_w denote the probability of 1 and w chip-interferences, respectively, between two users:

$$
\begin{aligned}
p_w &= \frac{1}{L} \cdot \frac{1}{|C|} = \frac{1}{L} \cdot \left\lfloor \frac{L-1}{w(w-1)} \right\rfloor^{-1} \\
p_1 &= \frac{\omega^2}{L} - w p_w .
\end{aligned}
\tag{3}
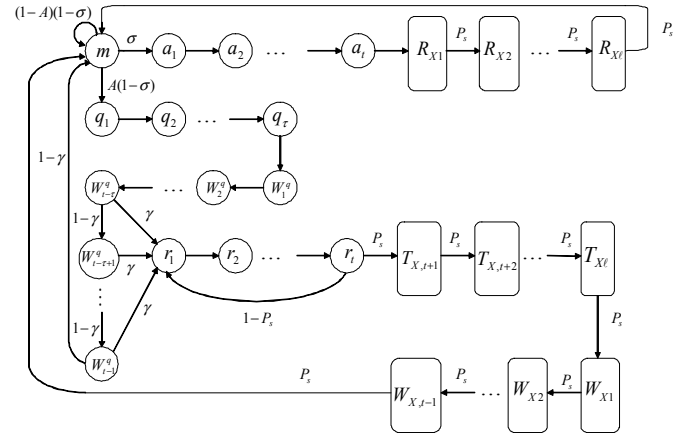$$

II. MATHEMATICAL MODEL

Fig. 1. State diagram of the proposed optical CDMA protocol.

The state diagram of the proposed protocol is shown in Fig. 1. Each state is labelled by its number of users. The states are defined as follows:

- *Initial state, $\{m\}$*. A user is in state m if its receiver is scanning across the codes. It takes one time slot to tune to a particular code. Once tuned to a particular code, if the station finds a request on that code (an event that occurs with probability σ), it proceeds to send an acknowledgment. If there is no request and there is an arrival (that occurs with probability A), the station enters the *requesting mode*. If there is neither a request nor an arrival, the station remains as is.
- *Requesting mode, $\{q_1, q_2, \ldots, q_\tau\}$*. If there is a message arrival and no request, the station proceeds to

send a connection request and tune its receiver to the code of the destination. It takes one time slot to tune a receiver, q_1. At the same time, the station sends repeated requests $\{q_1, q_2, \ldots q_\tau\}$ for τ time slots, where $\tau \geq 1$ is the timeout duration in time slots. After sending the last request, the station enters a waiting mode, $\{W_1^q, W_2^q, \ldots, W_{t-1}^q\}$, $t \geq \tau$, for $t-1$ time slots, till the reception of an acknowledgment from the destination. At the earliest, the station receives an acknowledgment (with probability γ) after a two-way propagation delay of t time slots from the first request q_1. In this case the station enters the *transmission mode*. Otherwise it stays in the waiting mode for the remaining $\tau - 1$ time slots. During each slot in the waiting mode, the station may receive an acknowledgment with probability γ. If no acknowledgment is received, the station is *timed out* and goes back to the initial state m.

- *Acknowledgment mode*, $\{a_1, a_2, \ldots, a_t\}$. The station enters these states after the arrival of a connection request. The station sends an Acknowledgment to the requesting station and waits for t time slots till it starts to receive the first packet. At this instant it enters the *reception mode*.
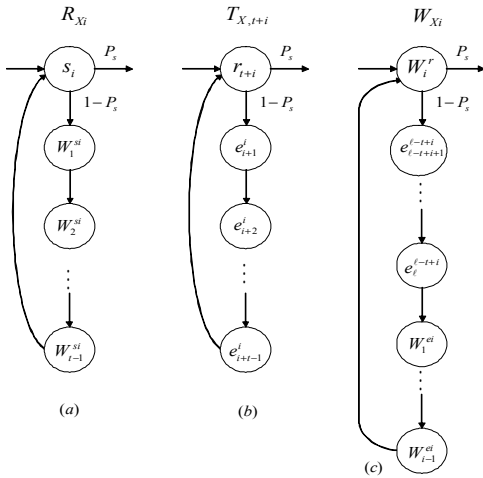

Fig. 2. (a) State R_{Xi}; (b) state $T_{X,t+i}$; (c) state W_{Xi}.

- *Reception mode*, $\{R_{X1}, R_{X2}, \ldots, R_{X\ell}\}$. The station enters state s_i in R_{Xi}, Figs. 1 and 2a, if it starts receiving packet i, $i \in \{1, 2, \ldots, \ell\}$. If the packet is received successfully, the station sends an Acknowledgment to the transmitting station and moves to s_{i+1} in $R_{X,i+1}$, otherwise it sends an ask-for-retransmission and enters a waiting mode $\{W_1^{si}, W_2^{si}, \ldots, W_{t-1}^{si}\}$, Fig. 2a, till the arrival of the retransmitted packet i. If the station is in state s_ℓ and a packet is received successfully, it goes back to the initial state m.
- *Transmission mode*, $\{r_1, r_2, \ldots, r_t, T_{X,t+1}, \ldots, T_{X\ell}\}$. The station is in state r_i, $i \in \{1, 2, \ldots, t\}$, if it is transmitting the first $t < \ell$ packets. After that the station receives an acknowledgment from the receiving station about the status of the first packet. If the

transmission is successful, the station transmits the next packet (enters state r_{t+1} in $T_{X,t+1}$), Figs. 1 and 2b. If it is not successful, however, the station goes back to state r_1 and start retransmitting the whole previous t packets. If the station is in state r_{t+i} of $T_{X,t+i}$, $i \in \{1, 2, \ldots, \ell - t\}$, Figs. 1 and 2b, it is transmitting packet $t+i$. After this transmission, it receives a status acknowledgment about packet $i+1$. If the status is positive (successful) the station enters state r_{t+i+1} in $T_{X,t+i+1}$. Otherwise it retransmits the last t packets (starting from packet $i+1$). That is it enters states $\{e_{i+1}^i, e_{i+2}^i, \ldots, e_{i+t-1}^i, r_{t+i}\}$ in Fig. 2b. If the station is transmitting the last packet ℓ (state r_ℓ in $T_{X\ell}$) and transmission $\ell - t + 1$ is successful, it enters a waiting mode, $\{W_{X1}, W_{X2}, \ldots, W_{X,t-1}\}$, Figs. 1 and 2c, to collect the status of the last $t-1$ packets. If a packet is unsuccessfully received, the station retransmits it and all subsequent packets and waits till a new status is received. That is, it enters the states shown in Fig. 2c (e denotes packet retransmission and W denotes slot waiting).

III. THEORETICAL ANALYSIS

Because of the complexity of the mathematical model given above, we use the method of equilibrium point analysis to measure the performance of the proposed protocol [5]. At an equilibrium point of the system, the expected number of users entering a state (in any time slot) is equal to that departing from it. Using this method of analysis, we will be able to figure out the steady state performance of the system by writing down the flow equation for each state. We start our analysis by assuming that $\ell \geq t$, where $t \geq \tau \geq 1$.

A. Transmission Mode

This mode involves states $\{r_1, r_2, \ldots, r_t\}$ and set of states $\{T_{X,t+1}, T_{X,t+2}, \ldots, T_{X\ell}\}$. From Figs. 1 and 2, we have written a set of flow equations, which (after some algebraic manipulations) can be reduced to:

$$r \overset{\text{def}}{=} \sum_{i=1}^{\ell} r_i = \ell r_1$$

$$W^r \overset{\text{def}}{=} \sum_{i=1}^{t-1} W_i^r = (t-1)r_1$$

$$e \overset{\text{def}}{=} \sum_{i=1}^{\ell-t} \sum_{j=i+1}^{i+t-1} e_j^i + \sum_{i=1}^{t-1} \sum_{j=\ell-t+i+1}^{\ell} e_j^{\ell-t+i}$$

$$= (1 - P_s)(t-1)(\ell - t/2)r_1$$

$$W^e \overset{\text{def}}{=} \sum_{i=1}^{t-1} \sum_{j=1}^{i-1} W_j^{ei} = (1 - P_s)(t-1)(t/2 - 1)r_1 \ . \quad (4)$$

B. Reception Mode

By writing the flow equations for the states in this mode and performing some algebraic simplifications, we are able to show that:

$$s \stackrel{\text{def}}{=} \sum_{i=1}^{\ell} s_i = \ell r_1$$

$$W^s \stackrel{\text{def}}{=} \sum_{i=1}^{\ell} \sum_{j=1}^{t-1} W_j^{si} = (1 - P_s)(t-1)\ell r_1 \ . \qquad (5)$$

C. Acknowledgment Mode

We write the flow equations for the states in the acknowledgment mode and simplify them to get:

$$m = \frac{P_s}{\sigma} r_1 \ . \qquad (6)$$

and

$$a \stackrel{\text{def}}{=} \sum_{i=1}^{t} a_i = P_s t r_1 \ . \qquad (7)$$

D. Requesting Mode

Again by writing the flow equations for the states in this mode, we get (after simplification):

$$q + W^q \stackrel{\text{def}}{=} \sum_{i=1}^{\tau} q_i + \sum_{i=1}^{t-1} W_i^q$$

$$= \left[t - 1 + \frac{1}{\gamma} \left(1 - (1-\gamma)^{\tau} \right) \right] A \frac{1-\sigma}{\sigma} P_s r_1 \ . \ (8)$$

Here γ denotes the probability that a station (in the requesting mode) gets an Acknowledgment. It can be evaluated by writing the flow equation into state r_1:

$$\gamma = 1 - \left[1 - \frac{\sigma}{A(1-\sigma)} \right]^{1/\tau} \ . \qquad (9)$$

The probability that a request is found by a scanning user, σ, is equal to the probability that another user is in states $q_1, q_2, \ldots,$ or q_τ:

$$\sigma = \frac{1}{N} \sum_{i=1}^{\tau} q_i = \frac{1-\sigma}{\sigma} A P_s \tau \frac{r_1}{N} \ , \qquad t \geq \tau \geq 1$$

This is a second order equation whose solution is easily found:

$$\sigma = \frac{1}{2} \left[\sqrt{u^2 + 4u} - u \right] \ , \qquad \text{where} \qquad u = A P_s \tau \frac{r_1}{N} \ . \tag{10}$$

E. Steady-State Throughput

The steady state throughput $\beta(N, A, t, \tau, \ell)$ is defined as the number of successfully received packets per slot:

$$\beta(N, A, t, \tau, \ell) = \sum_{i=1}^{\ell} s_i P_s = P_s(r_0) \ell r_1 \ , \qquad (11)$$

where r_0 denotes the number of transmitting users in a given slot:

$$r_0 = r + e = \left[\ell + (1 - P_s)(t-1)(\ell - t/2) \right] r_1 \ . \qquad (12)$$

Thus

$$\beta(N, A, t, \tau, \ell) = \frac{P_s(r_0) \ell r_0}{\ell + \left(1 - P_s(r_0) \right)(t-1)(\ell - t/2)} \ . \quad (13)$$

Here r_0 is determined so as the total number of users in all states is equal to N. That is r_0 is the solution of

$$N \left[\ell + \left(1 - P_s(r_0) \right)(t-1)(\ell - t/2) \right]$$

$$= r_0 \left[2t\ell \left(1 - P_s(r_0) \right) + (2t + 2\ell - 1) P_s(r_0) \right.$$

$$+ \frac{P_s(r_0)}{\sigma} + A(t-1)\frac{1-\sigma}{\sigma} P_s(r_0)$$

$$\left. + \left\{ 1 - \left[1 - \frac{\sigma}{A(1-\sigma)} \right]^{1/\tau} \right\}^{-1} P_s(r_0) \right], (14)$$

where

$$\sigma = \sigma(r_0) = \frac{1}{2} \left[\sqrt{u^2 + 4u} - u \right] \quad \text{and}$$

$$u = \frac{A P_s(r_0) \tau r_0}{N \left[\ell + \left(1 - P_s(r_0) \right)(t-1)(\ell - t/2) \right]} \ . \qquad (15)$$

IV. Numerical Results

The steady-state system throughput, derived above has been evaluated for chip-level receivers with different network parameters. Our results are plotted in Figs. 3–5. A packet size of $K = 127$ bits and a code weight of $w = 3$ are held fixed in all results. A chip time constraint $T_c = 0.254$ ns is imposed in all figures. A code length of $L = 31$ is chosen in all figures but Fig. 5. The above parameter selection ensures a slot size of $T_s = KLT_c = 1 \ \mu$s. Finally a message length of $\ell = 15$ packets and a timeout duration of $\tau = 1$ slot (or $\tau T_c = 1 \ \mu$s) are imposed in all figures.

In Fig. 3, the throughput has been plotted versus the number of users N for different propagation times $t \in \{2, 4, 6\}$ slots and same average activity $A = 0.5$. Similar trends of the curves can be noticed. There is always an optimum value of N that maximizes the throughput. Also the peak value changes slightly with the increase in propagation delay.

In Fig. 4 we have plotted the throughput versus the message length ℓ. Again an optimum value of ℓ always
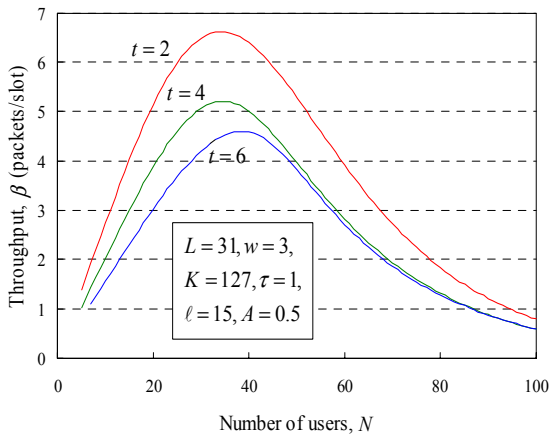
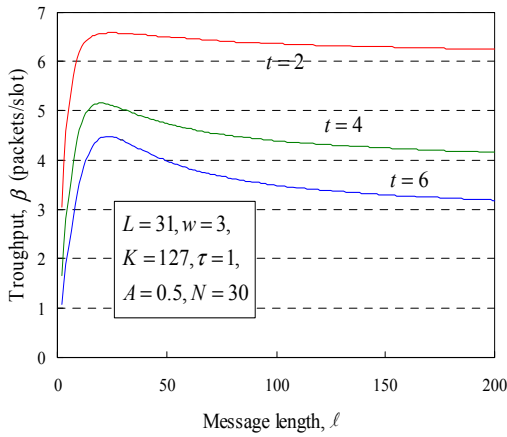Fig. 3. Throughput vs. number of users N for different interstation distances.

Fig. 4. Throughput vs. message length ℓ for different interstation distances.

exists. But the throughput reaches a saturation value as ℓ increases without limit. Indeed if users have messages of unlimited length to transmit, every user will be busy either transmitting or receiving packets. In this case, no change in the interference pattern happens.

In Fig. 5, the throughput has been plotted versus N for different code lengths L. Increasing L in this case enforces an increase to the slot size $T_s = KLT_c$. However, the propagation time t (in slots) will decrease for a fixed interstation distance $z = 600$ m. In this figure, we use a normalized throughput in packets/μs rather that in packets/slot, since the slot size is no longer fixed. It can be seen from Fig. 5 that for small number of users, it is better to use smaller code lengths and as the number of users increases, longer codes would give better throughput. Indeed, for small number of users, the interference among them is not that much. This leads to almost same number of successful packets per slot, independent of the code length. The use of longer codes here increases the slot size and decreases the number of successful packets per unit time. Thus it would be better to use shorter
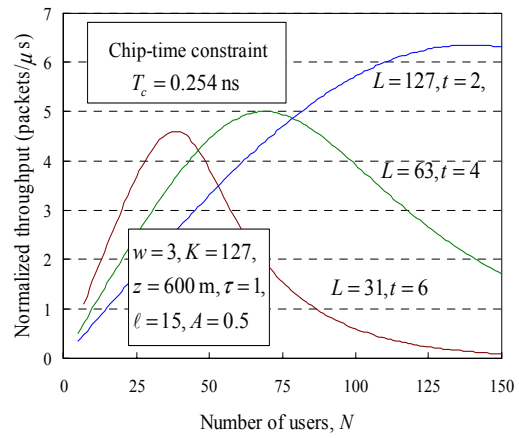

Fig. 5. Throughput vs. number of users N for different code lengths and a fixed chip time.

codes. However, for large number of users, the interference becomes too much to use short codes. In this case, most of the packets fail during transmission and users become busy retransmitting their failed data. Thus it would be better to use longer codes.

V. Concluding Remarks

A random-access protocol has been proposed for an optical direct-detection code-division-multiple access packet network. the steady-state system throughput (as a performance measure) has been derived based on the equilibrium point analysis technique. The effect of several design parameters on the system performance measure has been investigated and presented numerically for the case of chip-level receivers. Our results reveal that: (1) Optimum values (that maximizes the throughput) of the number of users always exist. (2) The throughput approaches a saturation value (close to its peak) as the message length increases without limit. (3) For small number of users, better throughput is achieved with short code lengths, whereas for large number of users, better throughput is achieved with longer codes.

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