Above the nominal limit performance evaluation of multiwavelength optical code-division multiple-access systems

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Abstract. We provide an analysis for the performance of a multiwavelength optical code-division multiple-access (MW-OCDMA) network when the system is working above the nominal transmission rate limit imposed by passive encoding-decoding operation. We address the problem of overlapping in such a system and how it can directly affect the bit error rate (BER). A unified mathematical framework is presented under the assumption of one-coincidence sequences with nonrepeating wavelengths. A closed form expression of the multiple access interference limited BER is provided as a function of different system parameters. Results show that the performance of the MW-OCDMA system can be critically affected when working above the nominal limit, an event that can happen when the network operates at a high transmission rate. In addition, the impact of the derived error probability on the performance of two newly proposed medium access control (MAC) protocols, the S-ALOHA and the $R^3T$, is also investigated. It is shown that for low transmission rates, the S-ALOHA is better than the $R^3T$, while the $R^3T$ is better at very high transmission rates. In general, it is postulated that the $R^3T$ protocol suffers a higher delay mainly because of the presence of additional modes. © 2009 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.3099613]

Subject terms: multiwavelength; optical code-division multiple access; fiber Bragg grating; multirate; overlapping coefficient; one-coincidence sequences; S-ALOHA; $R^3T$.

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1 Introduction

Lately, multiwavelength optical code-division multiple-access (MW-OCDMA) has received considerable attention as a multiple access scheme for optical local area networks due to its flexibility and diversity. In addition, multiaccess supporting multirate transmission using MW-OCDMA are now feasible due to the rapid evolution of fiber optic technology that offers ultrawide optical bandwidth capable of handling fast transmission rates. Toward this target, the bit-error-rate (BER) analysis of such a system is a crucial task.

MW-OCDMA has been proposed and discussed in numerous works. In addition, the BER has been derived and studied in detail. In Ref. 4, we have proposed a multirate MW-OCDMA system using fiber Bragg grating and variable processing gain (PG). The idea was to respect the total round-trip time for light from a data bit to go through the encoder. One of the key issues emphasized in Ref. 4 is the difference between passive optical CDMA and its electrical active counterpart. In fact, it has been argued that in active CDMA systems there is a one-to-one correspondence between the transmitted symbol duration and the PG. On the other hand, this one-to-one relation does not exist in passive optical CDMA systems. For instance, decreasing the bit duration will not affect the symbol duration at the output of the optical encoder. Therefore, for a fixed PG, increasing the link transmission rate beyond a given value, known as the nominal rate, leads to bit overlap at the output of the encoder. In Ref. 4, the general problem we have considered is how much we can increase the transmission rates of different classes of traffic beyond the nominal permitted rates to optimize performance to meet the quality of service (QoS) requirements. The QoS requirement has been taken to be the signal-to-interference ratio (SIR).

Although the SIR is considered to be a good QoS index, some network managers prefer the BER as a more reliable and exact QoS measure. In Refs. 2 and 5, the BER for the MW-OCDMA system has been presented in detail when the system works below the nominal limit. This means that sequences with ideal cross-correlation function of a maximum one were assumed. The probability of having one hit between two code sequences was obtained and the BER is derived using the binomial distribution. On the other hand, in Ref. 1, although the performance analysis of code sequences with arbitrary cross-correlation values is considered, the explicit equation of the probability of having more than one hit between two code sequences was not presented. Only the probability of having one and two hits was shown in Ref. 6 where a special family of nonideal optical orthogonal codes was analyzed.

In this work, we try to analyze the performance of the


**2 System Model**

Consider an MW-OCDMA communication network that supports $M$ users, sharing the same optical medium in a star architecture. A unified mathematical framework is presented in a way that the probability of having any number of hits between sequences can be obtained. We focus on the problem of overlapping in such a system and how it can mathematically affect the expression of the BER. Using this expression, the performance of the MW-OCDMA system is investigated in packet optical networks using two newly proposed medium access control (MAC) protocols, the R$^3$T and the S-ALOHA.

Following the introduction, the work is structured as follows. Section 2 introduces the system model. The effect of overlapping on frequency hits and interference quantification is discussed in Sec. 3. Section 4 presents the BER analysis assuming multiple access interference (MAI)-limited noise effect. Numerical results are covered in Sec. 5. Finally, the conclusion is presented in Sec. 6.

**MW-OCDMA system when the network is working above the nominal rate limit imposed by passive encoding-decoding operation. A unified mathematical framework is presented in a way that the probability of having any number of hits between sequences can be obtained. We focus on the problem of overlapping in such a system and how it can mathematically affect the expression of the BER. Using this expression, the performance of the MW-OCDMA system is investigated in packet optical networks using two newly proposed medium access control (MAC) protocols, the R$^3$T and the S-ALOHA.**

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**Fig. 1** The concept of overlapping among the bits, showing the effect of the overlapping coefficient $e_i$ on their transmission rate.

When the data rate increases beyond $R_n$, multibits will be coded during the time period $T_n$ and transmitted. At a given receiver, the decoder observes practically multicodes, which are delayed according to the transmission rate of the source, as shown in Fig. 1. When user $k$ transmits using rate $R_j > R_n$, it introduces a bit overlap coefficient $e_i$, according to which the new transmission rate is related to the nominal rate through the following equation,

$$R_j = \frac{G}{G-e_j} R_n,$$

where $0 \leq e_j \leq G-1$. This implies that $R^{(1)} \leq R_j \leq R^{(u)}$, where $R^{(1)} = R_n$ and $R^{(u)} = GR_n$ are the lower and upper data rates common to all users, respectively. Also, we assume that the system is chip synchronous and of discrete rate variation. Furthermore, all users transmit with equal power and have the same overlapping coefficient. In the example presented in Fig. 1, the concept of overlapping is illustrated among six bits of $G=5$ and the overlapping coefficient is $e_j=3$, which means that there are three chips in each OCDMA-coded bit that overlap with three chips of the other bits. This, in turn, augments the overall transmission rate of the users involved in this class from three bits after $3T_n$ to five bits. In general, the overlapping coefficient represents the number of overlapped chips among consecutive bits.

In Fig. 1 we define $\lambda_k^i$ as the wavelength of the $k$'th user at the $i$'th time slot, which is reflected by the $i$'th FBG. From Fig. 1, the optical bit stream can be seen to be serial-to-parallel converted to $v$ optical pulses. Because the bit $b_X^k$ from the $v$ bits is delayed by $\tau_{X} = X(G-e_j)$, this suggests that the channel model, as seen by the desired receiver, can be represented as a tapped delay line with tap spacing of $\tau_{i} = -(G-e_j)$ from the left and $\tau_{i} = (G-e_j)$ from the right. The tap weight coefficients $b_X^k \in \{0,1\}$ depend on whether the transmitted bit is zero or one.

Accordingly, the average cross-correlation function between two one-coincidence sequences has been obtained in Ref. 4 and is given by
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\[ \tilde{R}(G, e_j) = \frac{1}{2F}[G + (G + e_j)X - (G - e_j)X^2], \]

where

\[ X = \left[ \frac{e_j}{G - e_j} \right], \]

and \( F \) is the total number of available wavelengths from which the code is generated.\(^{10,11}\)

Although we have been able to study the performance of the multirate MW-OCDMA in Ref. 4, the work was based on the average of the cross-correlation function assuming one-coincidence sequences. In this work, we attempt to evaluate the performance of this system probabilistically in a way to obtain a closed form solution of the exact BER analysis of this system.

3 Multiple Access Interference Identification

As shown in Ref. 4, increasing the transmission rate will increase the overlapping coefficient, and therefore will induce more interference. Throughout this section we try to study and quantify the effect of overlapping for one-coincidence sequences with nonrepeating frequencies.\(^{10}\)

3.1 Overlapped Interference Sequences Identification

Consider an interferer user \( k \) with \((G, e_j)\) as the one presented in Fig. 1. During the autocorrelation process, we look only to the nominal period \( T_n \). Therefore, the interfering sequence as seen by the desired user’s receiver is shown in Fig. 2. In this figure, we notice many important aspects. The first one is that at every chip position, there is an interferring pattern that forms a sequence with different elements like \( S_1 = \{\lambda_0^k, \lambda_1^k, \lambda_2^k\} \). The second observation is that each interfering sequence is repeated multiple times at different chip positions.

**Lemma 1.** In an overlapped optical CDMA system, let an interferer with \((G, e_j)\). At the desired correlation receiver end, and during the nominal observation time period \( T_n \), the observed interfering sequences are subdivided into two groups. In the first group there are \( m_1 \) sequences of length,

\[ N_1 = \left\lfloor \frac{G}{G - e_j} \right\rfloor, \]

with

\[ m_1 = G - (G - e_j)\left\lfloor \frac{e_j}{G - e_j} \right\rfloor. \]

The remaining

\[ m_2 = (G - e_j)\left\lfloor \frac{e_j}{G - e_j} \right\rfloor - e_j, \]

sequences form the second group in which each sequence is of length

\[ N_2 = \left\lfloor \frac{e_j}{G - e_j} \right\rfloor. \]

**Proof.** Consider the wavelength index of the interfering wavelength at the \( i \)th time slot to be \( i \). Thus, the interfering code can be represented by a sequence of positive integers \( S = \{1, 2, 3, \ldots, G\} \). For an interferer with an overlapping coefficient \( e_j \), the desired user’s correlation receiver observes, at the \( i \)th time slot, a chain sequence \( S_i \) with \( S_j \subseteq S \).

Let \( \alpha_i \) to be the smallest element in \( S_i \). Notice that the step size in \( S_i \) is the bit delay \( \tau = (G - e_j) \). In addition, it is clear that the total number of distinct interfering sequences is \( T \).
Therefore, each interfering sequence \( S_i \forall i \in \{1, 2, \ldots, \tau\} \) can be completely characterized by \( \alpha_i \) and \( \tau \) as follows

\[
\alpha_i = \beta_i(n) \mod(\tau),
\]

where \( \beta_i(n) \) is the \( n \)th element in \( S_i \) and \( n \in \mathbb{N} \), the maximum of which is \( N_i \), which needs to be determined for every sequence.

The first \( \tau \) interfering sequences completely identify the overall interfering patterns, because the patterns are repeated every \( \tau \) time slots. Thus, we can write \( \alpha_i = i \forall i \in \{1, 2, \ldots, \tau\} \). Using Eq. (8), \( N_i \) can be written as

\[
N_i = \frac{\beta_i(n) - i}{\tau} + 1.
\]

\( S_1 \) possesses the highest length \( N_1 \), while \( S_2 \) has the smallest length \( N_2 \). Using Eq. (9), we define the length difference \( \Delta \) to be

\[
\Delta = N_1 - N_2 = \frac{\beta_i(N_j) - \beta_j(N_i) + \tau - 1}{\tau}.
\]

Knowing that \( \beta_i(N_j) - \beta_j(N_i) = 1 \), we obtain \( \Delta = 1 \). This means that the \( \tau \) different interfering sequences can be classified into two groups that differ between each other by a length difference of one. The first group of sequences with the highest length is imposed by the last overlapped bit of index \( X \) given in Eq. (3). The number of sequences in this group is given by

\[
m_1 = X \tau = G - (G - e_j) \left\lfloor \frac{e_j}{G - e_j} \right\rfloor,
\]

which proves Eq. (5). Using \( \beta_i(N_j) = X \tau + 1 \) in Eq. (9), we obtain

\[
N_1 = X + 1 = \left\lfloor \frac{G}{G - e_j} \right\rfloor,
\]

which proves Eq. (4). Obviously the remaining

\[
m_2 = m - m_1 = (G - e_j) \left\lfloor \frac{e_j}{G - e_j} \right\rfloor - e_j
\]

sequences have length

\[
N_2 = N_1 - 1 = \left\lfloor \frac{e_j}{G - e_j} \right\rfloor,
\]

which prove Eqs. (6) and (7), respectively.

Lemma 1 is very important in the sense that it enables us to quantify the effect of overlapping on the interference patterns at a given receiver.

### 3.2 Hits Quantification Due to Overlap

By observing Fig. 2, we can notice clearly that there are two different kinds of interfering sequences, \( S_1 = \{\lambda_1, \lambda_2, \lambda_3\} \) and \( S_2 = \{\lambda_1, \lambda_3\} \). In addition, \( S_1 \) is repeated three times and \( S_2 \) is repeated two times. Thus, to study the contribution of \( S_1 \) on the MAI, we need to compare it with the desired user wavelengths where it is present. Therefore, we need to compare \( S_1 = \{\lambda_1, \lambda_2, \lambda_3\} \) with \( d_1 = \{\lambda_0, \lambda_2, \lambda_3\} \). The same argument can be applied to the sequence \( S_2 \) where we need to compare \( S_2 = \{\lambda_1, \lambda_3\} \) to \( d_2 = \{\lambda_1, \lambda_3\} \).

Take for example \( S_1 \) and \( d_1 \). It is clear that there are up to three possible matching events between elements in \( S_1 \) and elements in \( d_1 \). Thus one element in \( S_1 \) can be in \( d_1 \), two elements in \( S_1 \) can be in \( d_1 \), or three elements in \( S_1 \) can match with three elements in \( d_1 \). Each of those events represents the number of hits the interfering sequence \( S_1 \) induces at the desired user’s receiver. The probabilities of those events are critical in deriving the BER of this system.

In general, consider an interfering sequence \( S_i \) and the desired sequence \( d_i \), both of them with length \( N_i \), where \( N_i \) can be obtained using Lemma 1. The probability that \( S_i \) causes \( i \) hits in the cross-correlation function is the probability that exactly \( i \) terms in \( S_i \) match with \( i \) terms in \( d_i \). If we are selecting \( N_i \) wavelengths at a time from a set of \( F \) possible wavelengths, then the probability of having exactly \( i \) wavelengths match with the sequence \( d_i \) of length \( N_i \) and \( N_i - i \) not matching can be written as

\[
P_S(i) = \binom{N_i}{i} \binom{F - N_i}{N_i - i} \forall 0 \leq i \leq N_i < F,
\]

where \( \binom{\cdot}{\cdot} \) is the binomial coefficient. All the permutations of a given combination \( s_i \) will result in \( i \) hits, assuming that there are exactly \( i \) wavelengths in \( s_i \) that match with \( i \) wavelengths in \( d_i \).

An illustration of Eq. (11) is shown in Fig. 3, where we plot the probability that the interfering sequence \( S_i \) causes \( i \) hits versus \( i \) for \( N_i = 14 \) [Fig. 3(a)] and \( N_i = 9 \) [Fig. 3(b)] and for different values of \( G \). In addition, we assume that \( F = 2G \). Notice that as the PG increases, although the probability of having a smaller number of hits decreases, the probability of a higher number of hits increases. This result is not like we expect, as in the case of nonoverlapped systems where the length of the interfering sequence is always one and increasing the PG will lead to a decrease in the probability of hits.

### 4 Multiple Access Interference Limited Bit Error Rate Analysis

Obviously, it is clear that due to the overlapping process, the assumption of one-coincidence sequences will not guarantee the upper bound on the number of hits to be one, as in classical nonoverlapped systems. In fact, the number of hits between two overlapped code sequences is related to the number of interference sequences and the size of each of those sequences. For example, if we return to the case studied and shown in Fig. 2, both interference sequences \( S_1 = \{\lambda_1, \lambda_2, \lambda_3\} \) and \( S_2 = \{\lambda_1, \lambda_3\} \) may cause up to five hits with different probabilities of occurrence. The importance of our work is to highlight those differences and to emphasize their effect on the performance evaluation of the system.
To evaluate the performance of the overlapped system, we need to find the probability that two overlapped codes have one, two, or \( p \) hits in their cross-correlation function, such that \( p \leq G \). We proceed by showing the case presented in Fig. 2. The interference pattern of this example can be simplified to two sequences of interference or hits \([H_1, H_2]\), which represent the number of possible hits caused by interference sequences \( S_1 \) and \( S_2 \), respectively. In our example, \( H_1 \) can take four possible values \( H_1 = \{0, 1, 2, 3\} \), and \( H_2 \) can take three possible values \( H_2 = \{0, 1, 2\} \) with different probabilities as revealed in Fig. 4.

Different possibilities of hits caused by both sequences:

\[
\begin{array}{cccc}
H_1 & H_2 & H_1 & H_2 \\
0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 \\
0 & 2 & 1 & 2 \\
\end{array}
\]

Fig. 4 Different possible hit configurations.

Fig. 3 The probability that the sequence \( S_i \) causes \( i \) hits in the cross-correlation function.
are presented in Fig. 4. According to Fig. 4, to compute the probability of having \( j \) hits from a given interferer, we have to find the probability of having \( H_1 + H_2 = s \). For example, the probability of having three hits is obtained as follows:

\[
q_3 = \text{Pr}(\{H_1 = 3 \cap H_2 = 0\} \cup \{H_1 = 2 \cap H_2 = 1\} \cup \{H_1 = 1 \cap H_2 = 2\}) = \frac{1}{(N_1 + 1)(N_2 + 1)}(P_{S_1}(3) \cdot P_{S_2}(0) + P_{S_1}(2) \cdot P_{S_2}(1) + P_{S_1}(1) \cdot P_{S_2}(2))
\]

\[
= \frac{1}{(N_1 + 1)(N_2 + 1)} \sum_j \{P_{S_j}(j) \cdot P_{S_2}(k)\},
\]

where \( N_1 = 3 \) and \( N_2 = 2 \) are the lengths of the sequences \( S_1 \) and \( S_2 \), respectively. In addition, the parameters \( j \) and \( k \) are chosen such that \( j + k = 3 \), \( \forall j = \{0, 1, 2, 3\} \), and \( k = \{0, 1, 2, 3\} \). The parameter \( i \) represents the number of cases satisfying \( j + k = 3 \).

Lemma 2. Consider two positive integer numbers \( i = \{0, 1, \ldots, N_1\} \) and \( j = \{0, 1, \ldots, N_2\} \) with \( N_1 < N_2 \). The number of possible couples \((i, j)\), \( \Lambda \), satisfying \( i + j = s \), \( \forall s = \{0, 1, \ldots, N_1 + N_2\} \), is given by

\[
\Lambda = \begin{cases} 
  s + 1, & \text{if } 0 \leq s \leq N_1 \\
  N_1 + 1, & \text{if } N_1 < s \leq N_2 \\
  N_1 + N_2 - s + 1, & \text{if } N_2 < s \leq N_1 + N_2 
\end{cases}
\]

Proof. To achieve \( s = i + j \), the general form of \((i, j)\) can always be written as \((i, s - i)\). Therefore, the problem under consideration is simplified to find the bounds of \( i \). Consider first that \( 0 \leq s \leq N_1 \). In this case, \( i \) can be incremented from zero to \( s \). This in turn shows that the number of possibilities is \( s + 1 \). On the other hand, if \( N_1 < s \leq N_2 \), \( i \) can be incremented from zero to \( N_1 \). Thus, the total number of possibilities is \( N_1 + 1 \). The last possible range of \( s \) is \( N_2 < s \leq N_1 + N_2 \). In this case, \( i \) must start with \( s - N_2 \) and be incremented until \( N_1 + 1 \). The total number of steps is thus \( N_1 + N_2 - s + 1 \), which completes the proof of the Lemma.

Using Lemma 2, we can generalize Eq. (12) to the case of any number of hits. Therefore, the probability of having \( s \) hits is given by

\[
q_s = \frac{1}{(N_1 + 1)(N_2 + 1)} \sum_{i=0}^{b} P_{S_1}(s-i) \cdot P_{S_2}(i),
\]

where

\[
(a, b) = \begin{cases} 
  (0, s), & \text{for } 0 \leq s \leq N_1 \\
  (0, N_1), & \text{for } N_1 < s \leq N_2 \\
  (s - N_1, N_2), & \text{for } N_2 < s \leq N_1 + N_2 
\end{cases}
\]

In addition, using Lemma 2, we can generalize the probability of having \( s \) hits given in Eqs. (14) and (15) to any number of interfering sequences. For example, assume that the number of interfering sequences is four. Thus, let \( i = \{0, 1, \ldots, N_1\}, \ j = \{0, 1, \ldots, N_2\}, \ k = \{0, 1, \ldots, N_3\}, \) and \( t \)

\[
\text{Fig. 5 The probability mass function of the number of hits for differ-}
\]

\[
\text{net values of the overlapping coefficient } e_o.
\]

\[
\text{Distribution of the number of hits when } G = 15, \ F = 18, \text{and for different values of } e_o. \text{It is clear that when the system works above the nominal limit, it may induce more than one hit, even if the codes used are one-coincidence codes. This in turn will drastically influence the probability of error of the system. Notice that, as we further increase the transmission rate, the probability of having larger numbers of hits becomes higher, thus increasing the MAI.}
\]

Having obtained in Eq. (16) the probability of \( s \) hits in the cross-correlation function, we can now compute the MAI-limited BER when the system is working above the nominal limit. Let \( Z, TH, \) and \( M \) denote the cross-correlation value seen by the desired receiver, the decision threshold, and the total number of simultaneous users in the system, respectively. Obviously, an error occurs whenever the transmitted data bit is zero, but the interference at the desired receiver results in \( Z > TH \). Thus, the probability of error is given by

\[
\]
Then, the probability of having length \( l \) interfering users that has a cross-correlation value \( j \). Therefore, we assume that the data bit zeros and ones are equiprobable.

Assume that \( S \) is the total number of interfering pulses in the cross-correlation function and that each interfering user may contribute up to \( S \) pulses in it. Let \( I_j \) to be the number of interfering users that has a cross-correlation value \( j \). Then, the probability of having \( \{I_1, I_2, \ldots, I_S\} \) interfering users follows a multinomial probability density function and it is given by

\[
Pr(I_1, I_2, \ldots, I_S) = \frac{(M-1)!}{(I_1^1! I_2^1! \cdots I_S^1)!} \left( \frac{1}{M-1} \sum_{j=1}^{S} q_j \right)^{1} \prod_{j=1}^{S} q_j^{I_j},
\]

where \( q_j \) is given in Eqs. (16) and (17). Using Eq. (19) in Eq. (18), we obtain

\[
P_e = \frac{1}{2} \sum_{I_1=0}^{TH-1} \sum_{I_2=0}^{[TH-1-l_1/2]} \cdots \sum_{I_S=0}^{[TH-1-S^{-1}]j_S/2]} Pr(I_1, I_2, \ldots, I_S).
\]

5 Numerical Results

Throughout this section, we try to study and discuss the previously derived equations using numerical evaluation. In addition, the impact of the exact BER analysis on the performance of two newly proposed MAC protocols, the S-ALOHA and \( R^2T \) protocols, is also investigated. For a complete and detailed discussion of both protocols, the reader is invited to refer to Refs. 7 and 8. Throughout this section, we assume that the total number of stations is \( M = 15 \), and the processing gain is \( G = 15 \).

In Fig. 6, we show the error probability as a function of the number of users \( M \) and for different values of the overlapping coefficient \( \epsilon \) when the number of available wavelengths is double the code length [Fig. 6(a)] and when the number of available wavelengths is equal to the code length [Fig. 6(b)]. The error probability increases when increasing \( M \), as expected. In addition, the error probability increases when increasing the transmission rate, therefore increasing \( \epsilon \). This is due to the increase in the probability of hits. Notice the importance of the number of available frequencies \( F \) on the performance of MW-OCDMA by observing that the probability of error shown in Fig. 6(a) for \( F = 2G \) is much lower than that presented in Fig. 6(b) where \( F = G \). This means that code families that provide a flexibility of choosing \( F \) like the one in Ref. 10 can offer better performance than that in which \( F = G \).

In the following simulation, we assume that the packet length under S-ALOHA protocol is \( L = 100 \) bits/time slot and the message is one packet. While equivalently, under the \( R^2T \) protocol the packet length is one bit/time slot and \( L = 100 \) designates the message length in packets. Note that there is a correspondence between \( A \) and \( P_e \). In S-ALOHA, when a terminal enters the backlogged mode, it cannot generate new packets until all the accumulated ones in the system’s buffer are retransmitted. Consequently, the offered traffic varies according to the retransmission probability \( P_e \).

Meanwhile, in \( R^2T \), the terminal in case of transmission failure retransmits the last unsuccessful \( r \) packets with the same transmission probability (user’s activity) \( A \), which varies the offered traffic. For S-ALOHA, we assume \( P_e = 0.6 \), whereas for \( R^2T \), we assume \( A = 0.6 \), the time out duration \( \tau_o = 1 \) time slot, and the two-way propagation delay is \( \tau_d = 2 \) time slots.

The throughput of both systems is presented in Fig. 7. In Fig. 7(a) we present the throughput versus the offered traffic using a detection threshold \( TH = G/2 \), while Fig. 7(b) shows the throughput when using the optimal detection.
threshold. The impact of the detection threshold is very obvious in the sense that there is a noticeable increase in the network throughput when using the optimal detection threshold.

It is clear that, for high overlapping coefficients, the $R^3T$ protocol exhibits higher throughput than the S-ALOHA protocol, while the throughputs of the S-ALOHA protocol are higher than that of the $R^3T$ when $\varepsilon$ is relatively small. For the S-ALOHA protocol, when using the optimal detection threshold, the system reaches its maximum throughput of eight packets per time slot for $\varepsilon < 10$. On the other hand, the $R^3T$ protocol reaches its maximum throughput of seven packets per time slot for $\varepsilon < 10$, as revealed by Fig. 7(b).

As the transmission rate increases, the throughput of the S-ALOHA decreases compared to that of the $R^3T$. This means that the $R^3T$ protocol can manage higher-rate users better than the S-ALOHA due to its efficient administration of erroneous packets using the “Round Robin” and the “Go-back-n” protocols.

In Fig. 8 we present the average packet delay versus the system throughput, operating under the two mentioned protocols using the optimal detection threshold as in Fig. 8(b), and a nonoptimal detection threshold as in Fig. 8(a), respectively. Here again we remark that the $R^3T$ protocol exhibits higher delay at low throughput for relatively small values of $\varepsilon$. On the other hand, the S-ALOHA protocol exhibits higher delay at low throughput for higher values of $\varepsilon$. Notice that when using the optimal detection threshold, the S-ALOHA achieves it maximum throughput with virtually no delay when $\varepsilon < 10$, while the $R^3T$ protocol takes around one time slot to achieve its maximum throughput. This is due to the use of the “Go-back-n” protocol.

By comparing the performance of the MW-OCDMA system under both protocols, we notice that for low transmission rates, the S-ALOHA is better than the $R^3T$, while the $R^3T$ is better at very high transmission rates.

6 Conclusion
An analysis of the performance of the MW-OCDMA network when the system is working above the nominal transmission rate limit is provided. A unified mathematical framework is presented under the assumption of one-coincidence sequences with nonrepeating wavelengths. A
closed form expression of the BER is derived. Results show that the performance of the MW-OCDMA system may be critically affected when working above the nominal limit. In addition, the impact of the derived error probability on the performance of two newly proposed MAC protocols, the S-ALOHA and the $R^2T$, is also investigated. It is shown that for low transmission rates, the S-ALOHA is better than the $R^2T$, while the $R^2T$ is better at very high transmission rates. However, in general, the $R^2T$ protocol suffers a higher delay because of the presence of additional modes.

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