Enhanced mathematical model for both throughput and blocking probability of optical burst switched networks

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Abstract. An enhanced mathematical model is introduced to study and evaluate the performance of a core node in an optical burst switched network. In the proposed model, the exact Poisson traffic arrivals to the optical burst switching (OBS) node is approximated by assuming that the maximum allowed number of arrivals to the OBS node, in a given time slot, is 2 (instead of $\infty$). A detailed state diagram is outlined to illustrate the problem, and then a mathematical model based on the equilibrium point analysis technique is presented. Two performance measures, namely, the steady-state system throughput and the average blocking probability, are derived from the model, which is built in the absence of wavelength conversion capability. Our proposed model is aided by a simulation work that studies the performance of an OBS core node under the assumption of Poisson traffic arrivals (the exact case) and calculates the steady-state system throughput. The results obtained from the proposed mathematical model are consistent with that of simulation when assuming Poisson traffic arrivals, and this consistency holds for a certain range of traffic load. The effect of varying different network parameters on the average blocking probability is discussed. © 2009 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.3204159]

Subject terms: optical burst switching (OBS); optical circuit switching (OCS); optical packet switching (OPS); just-in-time (JIT); just-enough-time (JET).

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1 Introduction

Optical burst switching (OBS) is a new switching paradigm that can support bursty traffic introduced by upper-layer protocols or high-end user applications. OBS can be considered as the gate through which the envisaged world of optical Internet will be conquered by implementing Internet protocols or high-end user applications. OBS can be considered as the gate through which the envisaged world of optical Internet will be conquered by implementing Internet Protocol (IP) software directly over a wavelength-division multiplexing (WDM) optical layer (IP/WDM). The idea of burst switching, first proposed by researchers in Refs. 1 and 2, emerges to combine the best of both optical circuit switching (OCS) and optical packet switching (OPS). The burst is the basic switching unit in OBS networks (OBSNs). The variability in the burst length from being as short as a packet to being as long as a session puts OBS as an intermediate solution between OCS and OPS. The performance evaluation of OBSNs has appeared in literature by several authors (e.g., in Refs. 3–11). Previous work carried out in OBS performance evaluation either used simulation or simply adopted the M/M/K/K model. This paper introduces an enhanced mathematical model based on equilibrium point analysis (EPA) by which one can easily measure the performance of an OBSN.

The architecture of an OBSN is fully explained in Ref. 12. The data carried in the burst result from the aggregation process of many packets (e.g., IP packets) carried out by an assembly node at the edge of the OBSN using the appropriate assembly algorithm.4,13–15 After being assembled at edge nodes (called ingress nodes), the data bursts go through the core network, which consists of core nodes that have the function of forwarding the data bursts (without going back to the electronic domain) until reaching their destination edge nodes (called egress nodes). Egress nodes then disassemble bursts back into packets, each of them to go to its destination. Many assembly algorithms have been proposed by researchers in the current literature.13,14,16 In addition, various burst-scheduling algorithms, which schedule the reservation process carried out by control packets, have been presented in Refs. 17–20.

Generally, the main idea beyond all OBS protocols is the separation of the header (carrying the control information) and the payload (carrying the data). Thus, the control packet (header) will be on a separate channel, called the control channel, while the burst (payload) remains on the data channel.

In this paper, the focus is on studying the performance of an OBS core node using either the JIT protocol21,22 or the JET protocol23 in the reservation mechanism. Both protocols are considered as one-way reservation protocols. The main difference between JIT and JET is that the former implies an immediate reservation strategy while the latter adopts a delayed reservation policy. Briefly, the control packet in JIT protocol is sent prior to the data burst by some offset time to reserve sufficient bandwidth immedi-
the node width reservation is done just before the data burst reaches the node and configure the switching fabric to route the upcoming data burst to the destined output port.

JET is a more efficient protocol where the control packet contains information about the offset time and the bandwidth reservation is done just before the data burst reaches the node (i.e., the reservation is delayed till the data burst reaches the node). Another difference between JIT and JET is in the release mechanism of the reserved bandwidth. In JIT, there is a release packet that is sent explicitly to release the reserved bandwidth, while in JET the control packet contains information about the burst length to allow the core node to release the reserved bandwidth implicitly after the burst departs the node. In both protocols, the data burst waits at the ingress node in an electronic buffer for an offset time equivalent to the total time needed by the control packet to be processed at each node. Figures 1 and 2 illustrate both JIT and JET, respectively.

To make the proposed model valid to be used for both JIT and JET protocols, it is obligatory to compensate for the difference between the two reservation schemes applied in both protocols. This difference can be modeled as an artificial increase in the burst length in the case of the JIT protocol, whereas no increase is introduced to the actual burst length in the case of JET protocol.

The aim of this paper is to introduce an enhanced mathematical model that studies the performance of the OBS core node under certain assumptions that will be stated in Sec. 2. Shalaby proposed a simplified model to study the performance of an OBS core node assuming Bernoulli distribution for arrivals per time slot, which proved to be a good assumption until a certain traffic load when compared to the simulation results, assuming that arrivals follow Poisson distribution. After this traffic load, the model turned out to be nonconsistent. In addition, it has another limitation where the maximum value for the average number of arrivals per time slot is 1. A fixed burst length was assumed, which is not realistic because the burst length depends on the assembly algorithm used by the ingress node to aggregate the packets in a burst. These two drawbacks will be handled in this paper, resulting in more consistent results for a wider range of traffic load. Moreover, the proposed model deals with the case of no wavelength conversion capability in the OBS node, unlike previous models that adopt the M/M/k/k queue to model the performance of the node in which full wavelength conversion is assumed. In our model, assuming the absence of wavelength conversion can be supported by the fact that adding such advanced techniques may raise the overall cost of the node to an unaffordable limit.

The remainder of this paper is organized as follows. In Sec. 2, we present the considerations on which the mathematical model is built. In Sec. 3, the construction of the state diagram that expresses the performance of the OBS node is provided. Section 4 is devoted to a theoretical study for the performance of an OBSN, where derivation of the steady-state system throughput is given. Section 5 provides the numerical results of the derived performance measure from both the proposed mathematical model and simulation. Finally, we give our conclusion in Sec. 6.

2 Preanalysis Model Considerations

The huge number of states required to express a network makes the trace of these states and deriving mathematical expressions governing the performance of this network a very difficult job. Consequently, the proposed model is derived under certain considerations that are introduced as follows:

1. The system is assumed to be slotted in time such that the time slot "T", is the smallest time unit. This will facilitate the construction of the state diagram provided that the time slot chosen is small enough to consider the system continuous in time. Fixed burst length (virtual) is also assumed where T = l/Ts and l is an integer value. This assumption proves to be accepted, which will be justified later in the simulation results.

2. In this paper, EPA is used such that number of users entering a certain state equal to the number of users departing it (i.e., number of served users in each state) is always constant.

3. We assume without loss of generality that all arrivals require the same output port, which can be justified by assuming that the outgoing traffic will be uniformly distributed on all output ports.

4. Because the time slot is very small (to consider the system is continuous in time), we can properly assume that the maximum number of arrivals per time slot is 2. Thus, we can approximate the Poisson distribution to be
A node considered is assumed to have that the OBS node considered has no wavelength conversion scenario is when the control packet fails to reserve a wavelength for its ensuing data burst. In either case, the data burst is assumed to be dropped and should be excluded when calculating the throughput.

The first blocking scenario (i.e., the control packet collision) can be resolved by adjusting the appropriate ratio between the control channel group (group of wavelengths dedicated to carry control packets only) and the data channel group (group of wavelengths used to serve data bursts) as proposed in Ref. 24. As a result, the only possible blocking scenario is when the control packet fails to reserve a wavelength for its ensuing data burst.

3 State Diagram Construction

In this section, an OBS network with \( l = 2 \) and \( w \geq 2l \) is considered. This case is chosen as an example to begin our discussion with because it is very simple, especially having a reasonable number of states. The state diagram in this case can be constructed as shown in Fig. 3. There are nine states in the state diagram. The transition probabilities are written in terms of \( X, Y, \) and \( Z, \) which are defined as functions of \( k \) in the following equations:

\[
X(k) = P_1 - (k - 1) \frac{P_1}{w} + \left\{ P_2 - (k - 1)^2 \frac{P_3}{w^2} - \frac{P_3}{w^2} \times \left[ (n-(k-1)) \right] P_2 \right\} #, \tag{1}
\]

\[
Y(k) = \frac{P_2}{w^2} \times \left[ (w-(k-2)) \right] P_2, \tag{2}
\]

\[
Z(k) = P_0 + \left( \frac{P_1}{w} \right) ^# + \left( \sqrt{\frac{P_2}{w^2}} \right) ^##, \tag{3}
\]

where \( k \) is the number of wavelengths used by the OBS node for transmission of bursts at the next state it will enter and \( # P_i = (n/i) \times ! \) is \( n \) permutation \( i. \) The number signs added above brackets indicate that terms inside them represent blocking cases. A single number sign means that a single arrival is blocked (not served), whereas double number signs mean that two arrivals are blocked.
The states shown in the state diagram in Fig. 3 can be categorized into five types:

1. Initial state \( \{m\} \): It represents the case when the OBS node is not serving any bursts at all (i.e., the node is idle). The OBS node is in this state, with probability \( m \). After staying a time \( T_s \) (in seconds) in the initial state, one of three events may occur. Either there are no arrivals to the node (an event with probability \( P_0 \)) or there is one arrival (an event with probability \( P_1 \)) or there are two arrivals (an event with probability \( P_2 \)). In the first case, the OBS node will stay at state \( m \) [with probability \( Z(0) \)], whereas in the second case, the node will enter state \( r_1 \). In the third case, when the two arrivals are requesting to use two different wavelengths [an event with probability \( P_3(w-1)/w \)] both will be served and the node will enter state \( r_2 \) [with probability \( Y(2) \)], while if the two arrivals are requesting to use the same wavelength (an event with probability \( P_2/w \)), one of them is blocked and the other is served by the node, which goes to state \( r_1 \). Clearly, the probability for the node to go from state \( m \) to state \( r_1 \) is \( P_1 + P_2/w \), which can be given briefly by \( X(1) \).

2. 1-\( \lambda \) states \( \{r_1^1, r_1^2\} \): It represents the case when the OBS node is currently using one of the available wavelengths as requested by the control packet. Now, let us consider the case when the node is currently staying at state \( r_1^1 \) and there are no arrivals after \( T_s \) (an event that occurs with probability \( P_0 \)), then it will enter state \( r_2^1 \), which means that the OBS node is currently serving the second time slot of a single arrival. Also, if the node is at state \( r_1^2 \) and there is one arrival that requests to use the same wavelength currently used by the node (an event that occurs with a probability \( P_1/w \)), the arrival will be blocked and the node will go to state \( r_2^2 \). Also, if the node is at state \( r_1^2 \) and there are two arrivals, both requesting to use the same wavelength currently used by the node (an event that occurs with a probability \( P_2/w^2 \)), then both arrivals will be blocked and the node will go to state \( r_2^2 \). Clearly, the probability for the node to go from state \( r_1^1 \) to state \( r_1^2 \) is \( P_0 + P_1/w + P_2/w^2 \), which can be given briefly by \( Z(1) \). In a different situation, when the node is at state \( r_1^2 \) and there is one arrival that requests to use another wavelength [an event that occurs with a probability \( P_2(w-1)/w \)], then the node serves both arrivals and enters state \( r_2^{11} \) (defined later). Also, when the node is at state \( r_1^2 \) and there are two arrivals, one of them requesting the same wavelength currently used by the node and the other requesting another wavelength [an event with a probability \( P_3 - P_3/w^2 - P_3(w-1)/w^2 \)], then the first will be blocked and the other will be served by the node that enters state \( r_2^{11} \). Briefly, the transition probability from \( r_1^1 \) to \( r_2^{11} \) can be given by \( X(2) \).

3. 2-\( \lambda \) states \( \{r_2^1, r_2^2, r_2^{21}\} \): It represents the case when the OBS node is currently using two of the available wavelengths. All the above discussion can be repeated in the same manner for any 2-\( \lambda \) state. For example, when the node is at state \( r_2^{11} \), then it is serving slot 2 of the first single arrival and slot 1 of the second single arrival. Suppose there are no arrivals to the node after staying \( T_s \) in \( r_2^{11} \), then the node finishes serving slot 2 of the first single arrival and enters state \( r_2^2 \) to serve slot 2 of the second single arrival.

4. 3-\( \lambda \) states \( \{r_2^{11}, r_2^{21}\} \): It represents the case when the OBS node is currently using three of the available wavelengths. For example, when the node is at state \( r_2^{11} \), then it is serving slot 2 of the first two arrivals and slot 1 of the second single arrival. Suppose there is a single arrival to the node after staying \( T_s \) in \( r_2^{11} \), then the node finishes serving slot 2 of the first single arrival and slot 1 of the new single arrival. In the second case, the new arrival requests to use a wavelength different from that used by the node [an event with probability \( P_3(w-1)/w \)], then the node will enter state \( r_2^{21} \) to serve slot 2 of the first single arrival and slot 1 of the new single arrival. In the second case, the new arrival requests to use a wavelength that is currently used by the node, then the new arrival will be blocked and the node enters state \( r_2^2 \) to serve slot 2 of the old arrival.

5. 4-\( \lambda \) states \( \{r_2^{21}\} \): It represents the case when the OBS node is currently using four of the available wavelengths.

Using the same manner in the above discussion, one can easily construct the rest of the state diagram. From the state diagram shown in Fig. 3, it is quite simple to write the flow equations as follows:

\[
\begin{align*}
m &= (m + r_2^1 + r_2^2)Z(0) \\
r_1^1 &= (m + r_2^1 + r_2^2)X(1) \\
r_2^1 &= (r_1^1 + r_2^{11} + r_2^{21})Z(1) \\
r_2^2 &= (m + r_2^1 + r_2^2)Y(2) \\
r_2^{11} &= (r_1^1 + r_2^{11})X(2) \\
r_2^{12} &= (r_1^1 + r_2^{11} + r_2^{21})Y(3) \\
r_2^{21} &= (r_1^1 + r_2^{11} + r_2^{21})X(3) \\
r_2^{22} &= (r_1^1 + r_2^{11} + r_2^{21})Y(4).
\end{align*}
\]

Adding the condition that the sum of all state probabilities equals to 1 is a must to make these equations solvable

\[m + r_1^1 + r_1^2 + r_2^1 + r_2^2 + r_2^{11} + r_2^{12} + r_2^{21} + r_2^{22} = 1.\]
3.2 State Diagram Construction in the General Case (for Any \( w \) and \( l \))

In this section, we will present a general method that can be applied to construct the state diagram for any values of \( w \) and \( l \).

First of all, we will consider a \( k \)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) where \( i_n,j_n,...,j_1 \in \{1,2,...,l\}, \quad j_n,j_{n-1},...,j_1 \in \{1,2,...,w\}, \quad k \in \{1,2,...,w+2\}, \quad n \in \{1,2,...,l+w\} \), \( k=\sum_{i=0}^{n} i \) and \( i_n \geq i_{n-1} \geq ... \geq i_1 \).

Three different scenarios may generate this state. We can categorize all the states found in the state diagram into the following three main families, depending on the scenario that generates it:

1. **Family (1):** It contains all the states having \( i_1=1 \) and \( j_1=1 \). That is, the node in these states is serving one new arrival. The previous states that may generate this state are either a \((k-1)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) or a \( k \)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) or a \((k+1)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) (only if \( k+1 \leq w \)).

   The transition probability is given by
   
   \[
   P_{u1} = Pr\{\text{one new arrival}\} \\
   \times Pr\{\text{the arrival selects an unused wavelength}\} \\
   + Pr\{\text{two new arrivals}\} \\
   \times [Pr\{\text{one arrival selects an unused wavelength}\} \\
   + Pr\{\text{both arrivals select the same unused wavelength}\}].
   \]

   The flow equation that describes this family is
   
   \[
   \rho_{k/l}^{n_1,...,j_1} = X(k) \left[ \rho_{k/l}^{n_1,...,j_1} + \frac{P_2}{w} \right] \\
   \times \left[ P_1 - (k-1) \frac{P_1}{w} \right] \\
   + \frac{P_2}{w} \times \left[ \frac{w-(k-1)}{w} \right] P_2.
   \]

   where \( X(k) \) is slightly different from the definition already stated in Eq. (1). From this point on, \( X(k) \) is defined as follows:

   \[
   X(k) = P_1 - (k-1) \frac{P_1}{w} + \left\{ P_2 - (k-1) \frac{P_2}{w} \right\} \\
   \times \left[ \frac{w-(k-1)}{w} \right] P_2.
   \]

   The term \( 1-[k/w] \) is added to make the definition of \( X(k) \) hold for the case \((w \leq 2l)\) in which some states are no longer present; thus, the term \( 1-[k/w] \) helps cancel the term \( P_2/w \times \left[ \frac{w-(k-1)}{w} \right] P_2 \) in \( X(k) \) corresponding to the transition probability of entering the omitted state, which is no longer present.

2. **Family (2):** It contains all the states having \( i_1=1 \) and \( j_1=2 \). That is, the node in these states is serving two new arrivals. The previous states that may generate this state are either a \((k-2)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) or a \((k-1)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) or a \( k \)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \). The transition probability is given by

   \[
   P_{u2} = Pr\{\text{two new arrivals}\} \\
   \times Pr\{\text{both arrivals select unused wavelengths}\}.
   \]

   The corresponding flow equation is thus
   
   \[
   \rho_{k/l}^{n_1,...,j_1} = Y(k) \left[ \rho_{k/l}^{n_1,...,j_1} + \frac{P_2}{w} \right] \\
   + \frac{P_2}{w} \times \left[ \frac{w-(k-1)}{w} \right] P_2.
   \]

   where \( Y(k) \) is already defined by Eq. (2).

3. **Family (3):** It contains all the states that are not belonging to family (1) or (2). That is, the node in these states is not serving new arrivals. The previous states that may generate this state are either a \( k \)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) or a \((k+1)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) (only if \( k+1 \leq w \)) or a \((k+2)\)-\( \lambda \) state \( \rho_{k/l}^{n_1,...,j_1} \) (only if \( k+2 \leq w \)).

   The transition probability is given by
   
   \[
   P_{u3} = Pr\{\text{no arrivals}\} + Pr\{\text{one new arrival}\} \\
   \times Pr\{\text{the arrival selects a used wavelength}\} \\
   + Pr\{\text{two new arrivals}\} \\
   \times Pr\{\text{both arrivals select used wavelengths}\}.
   \]

   The corresponding flow equation is thus
   
   \[
   \rho_{k/l}^{n_1,...,j_1} = Z(k) \left[ \rho_{k/l}^{n_1,...,j_1} + \frac{P_2}{w} \right] \\
   + \frac{P_2}{w} \times \left[ \frac{w-(k-1)}{w} \right] P_2.
   \]

   where \( Z(k) \) is already defined by Eq. (3).

4 Theoretical Analysis and Performance Measures Evaluation

In this section, a detailed discussion for the proposed mathematical model aided with an intensive mathematical analysis to evaluate the two performance measures [i.e., the steady-state system throughput and the blocking probability in the general case (for any \( l \) and \( w \))] is provided.

For simplicity, we begin our analysis by solving the flow equations for the special case of an OBSN with \( l=2 \) and \( w=2l \) presented previously in (4). Solving this system of equations exactly without any approximations to find expressions for the state probabilities \((m,r_1,r_2,...)\) in terms of \( P_0, P_1, P_2, \text{ and } w \) requires many algebraic manipulations, which is a very tiring problem to be performed math-
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ematically. Thus, a MATLAB code is written to solve this system of equations. This code gets $l$, $w$, $P_0$, $P_1$, and $P_2$ as inputs, and then it generates the flow equations that describe the state diagram corresponding to the input values using Eqs. (5)–(7). Finally, it provides the state probabilities as calculated from the solution of the generated flow equations.

After running this code many times while $l=2$ and assigning different values for $P_0$, $P_1$, $P_2$, and $w$, we obtain the state probabilities at each time. On the basis of the results obtained, a very important conclusion is reached

$$r_1^1 = r_2^1 = r_2^2 = r_2^{12} = r_2^{11}.$$

What is very exciting is that this conclusion holds for all values of $P_0$, $P_1$, $P_2$, and $w$ tried while holding $l$ fixed at 2. Even with varying $l$ to take other values (2, 3, 4,...), the conclusion is still valid. Thus, the conclusion reached can be generalized for any value of $l$. The rule governing this equality is that the OBS node in these states uses the same number of wavelengths ($k$) and number of single arrivals served by the node at different time slots are equal in the two states (i.e., $r_{i_{1_{n_1}}...i_{2_{m_2}}}$ and $r_{j_{1_{n_1}}...j_{2_{m_2}}}$) are said to be equal when $k = \sum_{i=1}^{n} f_i = \sum_{j=1}^{m} j_i$, $n = m$, $j_i = j_p$, where $i \neq p$.

The above approximation will reduce the three families in Eqs. (5)–(7) into two categories only described in Eqs. (8) and (9). The states will belong to the second category when there are two arrivals served by the OBS node in each time slot in these states (i.e., $j_i = j_{i+1} = \cdots = j_l = 2$). Otherwise, the states will belong to the first category

$$a_{k,n} = X(k)$$

$$b_{k,n} = Y(k)[b_{k-2,n-1} + a_{k-1,n} + b_{k,n}]$$

if $k = 2n \land n \neq 0$, (9)

where $a_{k,n}$ denotes a state probability in the first category with $k$ reserved wavelengths from the resources of the OBS core node, $n$ represents the number of events in which at least a single or two arrivals are successfully served by the OBS node in this state, and $n \in \{1, 2, \ldots, l \land w\}$. $k \in \{1, 2, \ldots, w\}$, and $k \neq 2n$. $b_{k,n}$ denotes a state probability in the second category with $k$ reserved wavelengths, $n$ represents the number of events in which two arrivals are successfully served by the OBS node $n \in \{0, 1, 2, \ldots, l \land w/2\}$, and $k = 2n$, where $b_{0,0}$ means idle state.

For example in our special case $b_{0,0} = m$, $a_{1,l} = r_2^1$, $a_{2,l} = r_2^2$, $a_{3,l} = r_2^3$, $a_{4,l} = r_2^4$, and $b_{1,l} = r_2^1$, $b_{2,l} = r_2^2$, $b_{3,l} = r_2^3$, $b_{4,l} = r_2^4$. Equations (8) and (9) can be written as follows:

$$a_{k,n} = X(k)$$

$$b_{k,n} = Y(k)[b_{k-2,n-1} + a_{k-1,n} + b_{k,n}]$$

if $k = 2n \land n \neq 0$, (9)

where $a_{k,n}$ denotes a state probability in the first category with $k$ reserved wavelengths from the resources of the OBS core node, $n$ represents the number of events in which at least a single or two arrivals are successfully served by the OBS node in this state, and $n \in \{1, 2, \ldots, l \land w\}$. $k \in \{1, 2, \ldots, w\}$, and $k \neq 2n$. $b_{k,n}$ denotes a state probability in the second category with $k$ reserved wavelengths, $n$ represents the number of events in which two arrivals are successfully served by the OBS node $n \in \{0, 1, 2, \ldots, l \land w/2\}$, and $k = 2n$, where $b_{0,0}$ means idle state.

Also, a recurrence relation can be computed for the second category until we reach $b_{0,0}(m)$.

$$b_{k,n} = \left(\prod_{i=1}^{n} \left[G \left[k - (2i - 1) - 1\right] - 1\right]\right) \times b_{0,0}.$$

Finally, we simply can calculate any state in the first category from (12) in terms of $m$ by substituting for the last term (enclosed by the oval shape) by Eq. (13) but with replacing $n$ by $\{k + \sum_{p=1}^{h}(-1)^{i(2h-2)}\}/2$. Imposing the condition that the summation of all state probabilities equals 1 in (14) and substituting for $b_{2n}$ and $a_{k,n}$ in terms of $m$, then taking $m$ as a common factor we can compute $m$ in terms of $A$, $l$, and $w$.

$$m + \sum_{n=1}^{l \land w/2} \left(\binom{l}{n} \times b_{2n}\right) + \sum_{n=1}^{l \land w} \left(\binom{w \land (2n-1)}{n} \times \binom{2n-k}{n-2k}\right) \times a_{k,n} = 1,$$

where $(n/l)$ is the number of states that have the same probability $b_{2n}$ and $(n/l) \times (2n-k)/2$ is the number of states that have the same probability $a_{k,n}$.

After the calculation of $m$, we can compute the value of any state probability. Consequently, the steady-state throughput $\beta(A,l,w)$ can be calculated in Eq. (15) by simply multiplying each state probability by the number of
reserved wavelengths in this state (i.e., the number of successful served bursts in this state)
\[
\beta(A,l,w) = \sum_{n=1}^{\lfloor w/(2n) \rfloor} 2n \times \binom{l}{n} \times b_{2n,n} \sum_{n=1}^{\lfloor w/(2n-1) \rfloor} k \times \binom{l}{n} \times \binom{n}{2n-k} \times a_{k,n}.
\]  
(15)

To calculate the average blocking probability \( P_B(A,l,w) \) for a given time slot, at an OBS core node, we have studied all possible cases, which can be summarized as follows:

1. There is only one arrival, and it is blocked.
2. There are two arrivals, and one of them is blocked.
3. There are two arrivals, and both are blocked.

In order to characterize the blocking cases, one must differentiate between two distinct situations. The first situation, when the node is at a state in the second category \( b_{2n,n} \) in which blocking may arise in two different scenarios, as follows:

1. If \( i_n \neq l \), an arrival is blocked with probability \( 1 \times \{(kP_1/w + P_2 - k^2P_2/w^2 - P_2/w^2 \times [1 - \lfloor k/(w-1) \rfloor]) 
\times (w-2kP_2/w^2) + 2 \times k^2P_2/w^2 \}. \)

2. If \( i_n = l \), an arrival is blocked with probability \( 1 \times \{(kP_1/w + P_2 - k^2P_2/w^2 - P_2/w^2 \times [1 - \lfloor k/(w-1) \rfloor]) 
\times (w-2kP_2/w^2) \}. \)

In the second situation, if the node is staying at a state in the first category \( a_{k,n} \), blocking may arise in three different scenarios:

1. If \( i_n \neq l \), an arrival is blocked with probability \( 1 \times \{(kP_1/w + P_2 - k^2P_2/w^2 - P_2/w^2 \times [1 - \lfloor k/(w-1) \rfloor]) 
\times (w-2kP_2/w^2) + 2 \times k^2P_2/w^2 \}. \)

2. If \( i_n = l \) and \( j_n = 1 \), an arrival is blocked with probability \( 1 \times \{(kP_1/w + P_2 - k^2P_2/w^2 - P_2/w^2 \times [1 - \lfloor k/(w-1) \rfloor]) 
\times (w-2kP_2/w^2) \}. \)

3. If \( i_n = l \) and \( j_n = 2 \), an arrival is blocked with probability \( 1 \times \{(kP_1/w + P_2 - k^2P_2/w^2 - P_2/w^2 \times [1 - \lfloor k/(w-1) \rfloor]) 
\times (w-2kP_2/w^2) \}. \)

On the basis of the previous discussion, the blocking probability \( P_B(A,l,w) \) can be written as follows:

\[
P_B(A,l,w) = m \times \frac{P_2}{w} + \sum_{n=1}^{\lfloor w/(2n) \rfloor} b_{2n,n} \times \left\{ \binom{l}{n} \times \binom{n}{2n-k} \times a_{k,n} \right\}.
\]

5 Simulation and Results

As mentioned in Sec. 2, the assumption of fixed burst length in the proposed model should be justified. To do so, a simulation work is performed using MATLAB in which a comparison between the steady-state system throughput (\( \beta \)) in two cases is made: the first assuming fixed burst length at \( l = 50 \) and \( w = 32 \), and the second assuming random burst length that follows Gaussian distribution as proposed in Ref. 4. The average value of the Gaussian distribution is set to \( l = 50 \) and \( w = 32 \). Results of simulation shown in Fig. 4 reveal that the steady-state system throughput is almost the same in both cases which justifies the assumption of fixed burst length in the proposed mathematical model.

In addition, another simulation work is made to study the OBSN performance [i.e., by calculating the average
system throughput \( (\beta) \), assuming exact Poisson traffic arrivals. The results of this simulation will be taken as a reference to compare the results of our proposed mathematical model with and check its range of consistency.

Figures 5–7 are concerned with the two performance measures of our model, namely, the steady-state system throughput \( (\beta) \) and the blocking probability \( (P_B) \). Initially, the steady-state system throughput \( (\beta) \) is shown in Fig. 5, where it is drawn against the average network traffic arrivals \( (A) \) in three different cases. First, the results assuming exact Poisson arrivals, which are obtained from simulation, are drawn. Second, the results of our proposed mathematical model, which are derived from Eq. (15), are also drawn. Finally, the results of the older mathematical model proposed by Shalaby,3 which assumes Bernoulli traffic arrivals, are added. Also, to clarify that our proposed model, unlike previous models, assumes the absence of wavelength conversion, we draw the average throughput calculated from the Erlang-B formula derived from the \( M/M/w/w \) model10 in which the number of servers equal the number of wavelengths and there are no places in the queue. In the Erlang-B formula, full server accessibility is assumed and, thus, it represents the case of full wavelength conversion capability in the node unlike our model.

Comparing the four curves, one can not that the system throughput \( (\beta) \) derived using our proposed mathematical model is approximately equal to that obtained from simulation (assuming Poisson arrivals), and this equality holds up to average traffic arrivals of 0.6. On the other hand, it can be noted that the throughput \( (\beta) \) derived from the older model proposed in Ref. 3 is equal to that obtained from simulation only up to average traffic arrivals of 0.12. In the case of \( M/M/w/w \) model, the throughput \( (\beta) \) exceeds that of simulation. This proves that our proposed model provides a much better approximation for the exact case when compared to the older model proposed and more accurate representation of the introduced exact case considering the absence of full wavelength conversion capability compared to the \( M/M/w/w \) model.10

The comparison also shows that our model outperforms the older model in terms of the range of consistency. Beyond the limit of consistency, the throughput obtained from our proposed model has larger values compared to the exact case (Poisson arrivals).

The reason behind this model collapse could be returned to a failure in the assumed arrival distribution in our model, which limits the maximum number of arrivals in a given time slot to two and thus fails to approximate the Poisson arrivals beyond the limit of consistency.

Considering the other performance measure derived from our model, the average blocking probability, Figs. 6 and 7 demonstrate the results obtained for the blocking probability calculated from Eq. (16). First, in Fig. 6, the blocking probability is plotted versus both the number of wavelengths and the burst length at certain number of arrivals per burst length which is denoted by \( \kappa \), where it can be easily evaluated from the equation \( \kappa=Al \).

It can be noted from Fig. 6 that the blocking probability decreases with the increase of either the number of wavelengths or the burst length. This is because when the number of wavelengths increases there will be a better chance for an upcoming burst to find its channel empty to reserve. Moreover, as the burst becomes longer \( (l) \) increases, the average number of arrivals per time slot decreases \( (A) \) decreases because more packets are assembled into the longer burst. The effect of this fact is considered by holding \( \kappa \) fixed at 30. Consequently, the blocking probability decreases with the increase of the burst length. The price paid by assembling more packets into a larger burst is the increase in packet latency. Thus, we have a trade-off between the blocking probability and the latency.

Figure 7 shows the relation between the number of wavelengths and the burst length at fixed blocking probabilities \{0.1, 0.15, 0.2,...\}. These curves result from the intersection of the surface drawn in Fig. 6, with surfaces representing constant values of the blocking probability. It can be implied from the curves that if the burst becomes longer the effect of increasing the number of wavelengths on the blocking probability is not noticeable. This can be noted from the larger spaces separating the curves at larger burst lengths as compared to the smaller spaces at smaller burst lengths. For instance, increasing \( w \) from 10 to 20 while \( l=50 \), decreases the blocking probability from \( P_{B1}=0.5458 \) to \( P_{B2}=0.4559 \). On the other hand, increasing \( w \) from 10 to 20 while \( l=200 \), only decreases the blocking probability from \( P_{B1}=0.1134 \) to \( P_{B2}=0.0906 \), which is not significant as compared to the first case. On the basis of the above discussion, one can easily conclude that the effec-
tiveness of increasing the number of wavelengths allocated for reservation is stronger for small burst lengths. For longer bursts, wavelength conversion would be a better solution to resolve the contention taking place at the OBS core node.

6 Conclusion
The enhanced mathematical model evaluates two performance measures of an OBS core node, namely, the steady-state system throughput and the blocking probability. Numerical results are presented at different values of network traffic and various network parameters. On the basis of the presented results, one can come up with the following conclusions:

1. In spite of the complexity of the proposed mathematical model equations, the model is still an easy and reliable way to get fast results by direct substitution with known network parameters instead of great efforts spent on simulation.
2. Throughput results show that our proposed model provide a good approximation for the exact case (Poisson arrivals) and outperforms the older model proposed in Ref. 3 in terms of range of consistency.
3. Results also reveal that the proposed model deals with the case of no wavelength conversion capability in the node, unlike older proposed models, such as the M/M/w/w model. This can be the real case due to the large costs needed to install such advanced technologies, which may increase the overall cost of the core network to an unacceptable limit.
4. Results of the blocking probability show that adding wavelength conversion capabilities to the node is only required for large burst lengths, while increasing the number wavelengths is adequate to resolve the contention between small competing bursts.
5. Because of the complexity of the EPA technique, it will be unreasonable to use it in further work because the state diagram will be huge, especially when we add buffering capabilities to the resources of the core node.

References


