Estimation and cancellation of multi-user interference in synchronous fiber-optic PPM-CDMA system using Manchester coding

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Abstract

A multi-user interference estimation and cancellation technique is proposed for direct-detection fiber-optic code division multiple-access communication systems employing pulse-position modulation. In addition, Manchester codes are used in signaling the transmitted data to further improve the bit-error rate (BER). The multi-user interference of any user is estimated with the help of properties of modified prime code sequences. The estimated interference is canceled out from the received signal after the photo-detection process. We have used PIN photo-detector in our proposed system. An upper bound on the BER for the proposed system is derived and compared with a lower bound on the BER for the system without cancellation. In the presence of multiple users interference (MUI) and the Poisson shot noise model, our results clearly indicate that the performance, in terms of the BER, of the proposed system is significantly improved compared with that of the system without cancellation. The effect of thermal, dark current and surface leakage noise is insignificant compared to MUI and thus will not be considered in our calculation of BER. © 2001 Published by Elsevier Science Ltd.

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1. Introduction

The studies of fiber-optic code division multiple-access (CDMA) communication systems have gained substantial interest in the recent years [1–13]. With the vast bandwidth available, optical fibers are attractive media for spread spectrum communications. Synchronous CDMA systems have some advantages in local area networks over their asynchronous counterparts [6]. For example a synchronous CDMA system can accommodate a greater number of simultaneous users than an asynchronous CDMA system for a given bit-error rate (BER). Furthermore, the number of possible subscribers (available code sequences) is also greater in the case of synchronous CDMA. However, a master clock is required for synchronous CDMA systems.

Both pulse position modulation (PPM) and on-off keying (OOK) CDMA schemes are two popular modulation techniques in optical CDMA communication systems. PPM-CDMA outperforms OOK-CDMA in terms of power usage [1]. Furthermore, PPM-CDMA systems can accommodate any number of simultaneous users by increasing the pulse-position multiplicity while keeping the average power unchanged [13]. Both optical orthogonal codes [7] and modified prime code sequences [6] have been extensively used as signature codes in optical CDMA. Modified prime code sequences are well used in many interference cancellation systems [1,5]. This is because they exhibit what is called the grouping property, where all the sequences in this code are classified into different groups. Sequences from the same group have complete orthogonality, while those from different groups have incomplete orthogonality, with a constant cross-correlation of one.

Optical CDMA systems suffer from multiple-user interference due to the incomplete orthogonality of its codes. Several proposals to minimize this interference have appeared in literature. Lin and Wu [3] have proposed a synchronous FO-CDMA using adaptive optical hardlimiter with modified prime sequences as signature codes. In [4], Ohtsuki has proposed using an electrooptic switch in addition to the optical hardlimiters in optical OOK-CDMA systems. In [1], Shalaby has utilized the grouping property of modified prime sequences as a means of interference cancellation in optical PPM-CDMA systems. Due to the implementation complexity of the latter system, Shalaby [5] has proposed another
interference cancellation for OOK-CDMA systems by keeping one code in each group of the modified prime code sequences unallocated to any user. This unused code can be utilized to provide an estimate to the interference, whose effect can in turn be removed from the received signal. In [12], Lin et al. have proposed using random Manchester codes as a means of interference reduction in optical OOK-CDMA systems.

The aim of this paper is to extend the method introduced in Ref. [5] for the optical PPM-CDMA systems and analyze its overall performance. In the analysis, we have used Manchester codes in signaling the transmitted data for further improvement of the bit-error probability performance.

The remainder of this paper is organized as follows. In Section 2, we present a brief description of properties of modified prime code sequences and the optical PPM-CDMA system. The derivation of bit-error probability of the proposed system is given in Section 3. In Section 4, we compare the bit-error probabilities of the proposed system to that of both the PPM-CDMA system without cancellation and the system proposed in [1]. Finally the conclusion is given in Section 5.

2. Optical PPM-CDMA system description

The spreading sequences used in our analysis are the modified prime code sequences, which are time-shifted versions of prime sequence codes [13]. For any given prime number \( p \), there are \( p^2 \) code sequences that can be generated. Each code sequence has a weight \( p \) and a length \( p^2 \). The codes are divided into \( p \) groups where each consists of \( p \) different codes. The cross-correlation \( (C_{mn}) \) between codes \( m \) and \( n \), \( m, n \in \{1, 2, \ldots, p^2\} \), is given by

\[
C_{mn} = \begin{cases} 
  p & \text{if } m = n, \\
  0 & \text{if } m \text{ and } n \text{ share the same group and } m \neq n, \\
  1 & \text{if } m \text{ and } n \text{ are from different groups.}
\end{cases}
\]

The \( M \)-ary PPM signaling format used in the proposed system is shown in Fig. 1. Each symbol is represented by a train of pulses spread in one of the \( M \) disjoint time slots. The width of each slot is \( \tau \) seconds out of a transmission time frame of \( T \) seconds. Each slot consists of \( p^2 \) chip with chip time, \( T_c \). In a single time frame, each user is allowed to transmit only one of the \( M \) symbols. More than one user, however, can transmit the same symbol in a time frame. Each user is pre-assigned a unique spreading sequence (modified prime sequence code). When a user transmits a symbol, the unique code sequence of the intended user will occupy the corresponding slot.

The model for an optical PPM-CDMA transmitter is shown in Fig. 2. In the transmitter, each information source represents one user that transmits \( M \)-ary continuous data symbols. Since modified prime code sequences are the signature used, therefore there are altogether \( p^2 \) available users. These \( M \)-ary data symbols will be fed into the optical PPM encoder that modulates the pulse position of the injected laser based on the symbol sent. For example, in 8-ary communication, when symbol “011” is sent, the laser is pulsed on at the first chip of the fourth slot (as illustrated in Fig. 2). Then the output laser pulse is converted into the assigned spreading sequence by the optical CDMA encoder. The optical CDMA encoder unit spreads the initial laser into a specific train of output pulses. The train of output pulses is the address or signature of the intended user. For example if user 1 wants to send a symbol to user 2, the spreading sequences transmitted by user 1 will be the signature of user 2. Therefore, in this system, we are assuming that each user in the system knows exactly what is the address of each other user. The spread signal of all active users in the system is then added together and transmitted over the optical channel.

To further improve the bit-error probability, we have used the Manchester coding scheme as follows. The optical pulses for the spreading sequences in the first \((p + 1)/2\) groups of users (out of \( p \) groups) are signaled in the first half-chip intervals while the remaining \((p - 1)/2\) groups of users are using the second half-chip intervals. This coding scheme will ensure that the 2 groups of users will not interfere with each other and thus will help to reduce multi-user interference.

Fig. 3 shows a typical optical PPM-CDMA receiver without interference cancellation. At the receiver, an optical tapped delay line correlates the multiplexed signal with the spreading sequence of the intended user. The optical tapped delay line is a set of optical delay lines inversely matched.
to the pulse spacings. When the desired optical sequence passes through the correlator, the output light intensity traces the correlation function of the sequence. The amount of delays is not only dependent on the spreading sequence but also on the positions of the marks within the chip intervals. The optical signal is then converted to an electrical signal using a photo-detector. The electrical signal that is proportional to the photon counts will be integrated at every slot interval (i.e. every \( i \tau \) interval for \( i = 1, 2, \ldots, m \)). The electrical signal is integrated from \( t - T_c \) to \( t - T_c/2 \) for user whose spreading sequence is signaled in the first half-chip interval and \( t - T_c/2 \) to \( t \) for user whose spreading sequence is signaled in the second half-chip interval. The photon count of each time slot will be passed to the decision mechanism, after which the slot with the highest photon count is declared as the symbol sent for that particular time frame.

3. Optical PPM-CDMA with interference estimation

The performance of the optical PPM-CDMA system is affected by the interference from active users that are not in the same group as the desired user (i.e. the cross-correlation between users from different groups is equal to 1). The effect of this interference can be reduced by first finding an estimate to the interference and then removing this estimated value from the received data before passing it to the decision mechanism. To provide an estimate of the interference we apply a similar approach as that is used in OOK-CDMA [5].

In the system used in Ref. [5], the last code in each group is not assigned to any user and is reserved for multiple-user interference (MUI) estimation at the receiving end. This code sequence is assumed to be known to all users in the same group. The total number of subscribers is thus limited to \( p^2 - p \). Out of this number, we assume that there are \( N \)
active (simultaneous) users and the remaining \( p^2 - p - N \) users are idling.

To calculate the bit error probability, we first assume that each user is assigned a unique modified prime code sequence of length \( p^2 \). Let the spreading sequence of the \( k \)th user be \((d_{k,0}^{(a)}, \ldots, d_{k,-1}^{(a)})\), where \( d_{k,i}^{(a)} \in \{0,1\} \) and the periodic spreading waveform can be written as

\[
d^k(t) = \sum_{i=-\infty}^{\infty} d_{k,i}^{(a)} P_{T_c/2}(t - S_k \times T_c/2 - iT_c),
\]

(1a)

where \( d_{k,i}^{(a)} p^2 = d_{k,i} \) for all integers \( i \), \( T_c = \tau / p^2 \) is the chip time and \( P_{T_c/2}(\cdot) \) is a rectangular pulse of duration \( T_c/2 \), defined by

\[
P_{T_c/2}(t) = \begin{cases} 1, & 0 < t < T_c/2, \\ 0, & \text{otherwise} \end{cases}
\]

(1b)

The effect of Manchester coding is represented by the variable \( S_k \) in Eq. (1a). The variable \( S_k \) is defined as

\[
S_k = \begin{cases} 0, & \text{when the } k \text{th user is signaled in the first half-\textit{chip} interval}, \\ 1, & \text{when the } k \text{th user is signaled in the second half-\textit{chip} interval}. \end{cases}
\]

(1c)

Let us assume that the \( k \)th information source generates the data symbol \( \{b_j^{(k)}\}_{j=-\infty}^{\infty} \), where \( b_j^{(k)} \in \{0,\ldots,M-1\} \). This sequence will modulate the position of a laser pulse so that the output of the optical PPM encoder can be written as

\[
b^k(t) = \sum_{j=-\infty}^{\infty} \lambda_n P_{T_c}(t - b_j^{(k)} \tau - jT_c),
\]

(2)

where \( \lambda_n \) is the signal photon rate (which we assumed to be constant for all transmitters) and \( T \) is the PPM time frame previously defined. At the input of the receiver’s optical correlator, the total optical power can be expressed as

\[
s(t) = \sum_{k=1}^{N} b^k(t) d^k(t),
\]

(3)

where \( N \) is the number of simultaneous users.

We define a column vector \( V_n \) of size \( M \) for user \( n \). If this user sends symbol \( i \in \{0,1,\ldots,M-1\} \), then all entries in \( V_n \) will be equal to zero except for the \( i \)th entry. That is

\[
V_n = \begin{bmatrix}
V_{n,0} \\
V_{n,1} \\
\vdots \\
V_{n,i} = 1 \\
\vdots \\
V_{n,M-1} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
\end{bmatrix}.
\]

(4)

We also define \( H_{j,n}, j \in \{0,1,\ldots,M-1\}, n \in \{1,2,\ldots,p^2\} \), as follows:

\[
H_{j,n} = \begin{cases} 1, & \text{if user } n \text{ is sending symbol } j, \\ 0, & \text{else} \end{cases}
\]

Thus,

\[
\sum_{j=0}^{M-1} \sum_{n=1}^{p^2} H_{j,n} = N.
\]

Without loss of generality, let us assume that the user 1 is the desired user and the random variable \( T \) be the number of active users in the first group, we have

\[
T = \sum_{j=0}^{M-1} \sum_{n=1}^{p^2} H_{j,n}.
\]

(6)

The probability distribution of the random variable \( T \) given user 1 is active for any \( t \in \{t_{\min}, t_{\min+1}, \ldots, t_{\max}\} \), where \( t_{\min} = \max\{1, N + 2p - p^2 - 1\} \) and \( t_{\max} = \min\{N, p - 1\} \) can be expressed as [5]

\[
P_T(t) = \begin{pmatrix} p^2 - 2p + 1 \\ N - t \end{pmatrix} \begin{pmatrix} p - 2 \\ t - 1 \end{pmatrix}.
\]

(7)

We define another random variable \( R \) for the number of active users from group 2 up to group \((p + 1)/2\). The probability of this random variable for any \( r \in \{r_{\min}, r_{\min+1}, \ldots, r_{\max}\} \), where \( r_{\min} = \max\{0, N - t - (p^2 - 2p + 1)/2\} \) and \( r_{\max} = \min\{(p^2 - 2p + 1)/2, N - t\} \) given that \( T = t \) can be expressed as

\[
P_{R|T}(r|t) = \begin{pmatrix} p^2 - 2p + 1/2 \\ r \end{pmatrix} \begin{pmatrix} p^2 - 2p + 1/2 \\ N - t - r \end{pmatrix}.
\]

(8)

The block diagram of the proposed receiver is shown in Fig. 4, where the received signal at point A is split into two equal signals using a \( 1 \times 2 \) optical splitter. Signal at point A is from the optical channel where signals from all transmitters are multiplexed together, therefore, the signal at point A will consist of both desired signal and multiple user interference. The signal at point B (i.e. the upper branch) is correlated with the signature code sequence that characterizes the desired user using optical matched filter. The correlator output at point C is then converted to an electrical signal using a photo-detector. It is at the photo-detector where shot noise, dark current and surface leakage current noise are introduced. The electrical signal at the output of the photo-detector will be integrated and sampled as described in the previous section. The output of the sampler at point E is proportional to the photon count (denoted by \( Y_{1,j} \)) collected over the \( j \)th slot duration. Each \( Y_{1,j} \) is a conditional Poisson random variable. The signal at point F is directed to the lower branch, where the interference estimation process is accomplished. The signal at point F is correlated with the last code sequence in the desired user’s group, which was preserved a priori. Since the last code sequence in each group is left
are given by both code sequences are from the same group. The photon difference in the upper and lower branch will be the same since of modi, ed prime code sequences, the multiple-user interference is unused, therefore the signal at point G (i.e. output of optical matched filter at lower branch) will only be composed of multiple-user interference. Based on the grouping property of modified prime code sequences, the multiple-user interference in the upper and lower branch will be the same since both code sequences are from the same group. The photon count collected over the jth slot duration from this branch will be denoted by \( Y_{p,j} \). This photon count, \( Y_{p,j} \), from the interference is then subtracted from the photon counts \( Y_{1,j} \) in the main branch to produce the signal \( \tilde{Y}_{1,j} \) at point J. Therefore, effectively the signal that is left at point J is only the desired signal (i.e. without multiple-user interference). The decision on the transmitted symbol is accomplished in the main branch by selecting the index \( j \in \{0, 1, 2, \ldots, M - 1\} \) where \( \tilde{Y}_{1,j} \) is maximum.

Let the collection of the photon counts \( (Y_{n,0}, Y_{n,1}, \ldots, Y_{n,M-1}) \) be denoted by the Poisson random vector \( \mathbf{Y}_n \). The mean vectors \( \{Z_n\}_{n=1}^p \) for the photon count vectors \( \{\mathbf{Y}_n\}_{n=1}^p \) collected by the users sharing the same group with user 1 are given by

\[
Z_n = QpV_n + Q1. \tag{9}
\]

where as the interference random vector \( \mathbf{I} \) is given by

\[
\mathbf{I} = \sum_{n=p+1}^{[p+1)/2]} \mathbf{V}_n. \tag{10}
\]

Here, \( Q \) denotes the average received photon count per pulse. The average photon count due to the interference \( \mathbf{I} \) is the same for all users in one group. Given \( T = t \) and \( R = r \), it is easy to check that \( \mathbf{I} \) is a multinomial random vector with probability

\[
P_{\mathbf{I},T,R} (l_0, l_1, \ldots, l_{M-1} | t, r) = \frac{1}{M!} \frac{r!}{l_0!l_1! \cdots l_{M-1}!}, \tag{11}
\]

where \( \sum_{i=0}^{M-1} l_i = r \). To reduce the interference we construct the vector \( \tilde{\mathbf{Y}}_1 \) as follows:

\[
\tilde{\mathbf{Y}}_1 = \mathbf{Y}_1 - \mathbf{Y}_p. \tag{12}
\]

The decision to select the received symbol by user 1 is as follows. Symbol \( i \) is declared to be the correct one if \( \tilde{Y}_{1,i} > \tilde{Y}_{1,j} \) for every \( j \neq i \). An upper bound on the bit-error probability can be derived as follows:

\[
P_b = \frac{M}{2(M-1)} \sum_{t=0}^{t_{\text{max}}} \sum_{r_{\text{min}}} p_{\mathbf{I}, T} (t) p_{\mathbf{R} | \mathbf{T}} (r | t), \tag{13}
\]

where

\[
P_{\mathbf{I}, T} (t) = \sum_{i=0}^{M-1} \Pr \{ \tilde{Y}_{1,i} > \tilde{Y}_{1,i}, \text{some } j \neq i | T = t, R = r, V_{1,i} = 1 \} \]

\[
\Pr \{ V_{1,i} = 1 \}. \tag{14}
\]

For equally likely data symbols, we get

\[
P_{\mathbf{I}, T} (t) = \sum_{i=0}^{M-1} \Pr \{ \tilde{Y}_{1,i} > \tilde{Y}_{1,0}, \text{some } j \neq 0 | T = t, R = r, V_{1,0} = 1 \} \]

\[
\leq (M-1) \sum_{I} p_{\mathbf{I}, T, R} (l | t, r) \Pr \{ \tilde{Y}_{1,1} > \tilde{Y}_{1,0} | T = t, R = r, I = l, V_{1,0} = 1 \} \]

\[
\leq \sum_{I} p_{\mathbf{I}, T, R} (l | t, r) 0(t, I), \tag{15}
\]

where \( \theta(t, I) = (M-1) \Pr \{ \tilde{Y}_{1,1} > \tilde{Y}_{1,0} | T = t, R = r, I = l, V_{1,0} = 1 \} \).

By using the Chernoff bound, \( \theta(t, I) \) can further be upper bounded as

\[
\theta(t, I) = (M-1) \Pr \{ Y_{1,1} - Y_{p,1} > Y_{1,0} - Y_{p,0} | T = t, R = r, I = l, V_{1,0} = 1 \} \]

\[
\leq (M-1) E \{ z^{[Y_{1,1} - Y_{p,1} - Y_{1,0} + Y_{p,0}] | T = t, R = r, I = l, V_{1,0} = 1} \}, \tag{15}
\]

where \( E \{ \cdot | \cdot \} \) denotes the conditional expectation operator. Performing the last expectation, we obtain

\[
\ln \theta(t, I) \leq \ln(M-1) - Q_l(1-z) - Q_l(1-z^{-1}) - Q(p + l_0)(1-z^{-1}) - Ql_0(1-z). \]
Setting $z = 1 + \delta$, $\delta > 0$, we obtain the following upper bound on the last equation:

$$
\ln \theta(t, 1) \leq \ln(M - 1) - Q l_1(\delta - \delta^2) - Q(p + l_0)(\delta - \delta^2) - Q l_0(-\delta)
$$

$$
= \ln(M - 1) + Q l_1\delta^2 - Q p\delta + Q(p + l_0)\delta^2
$$

or

$$
\theta(t, 1) \leq (M - 1) \exp[-Q(p\delta - (p + l_0 + l_1)\delta^2)]. \quad (16)
$$

Searching for the tightest $\delta$, we get

$$
\delta = \frac{p}{2(p + l_0 + l_1)}. \quad (17)
$$

Hence

$$
\theta(t, 1) \leq (M - 1) \exp\left[-\frac{Q p^2}{4(p + l_0 + l_1)}\right] \quad (18)
$$

and the required upper estimate on $P_{E}^{t,r}$ reduces to

$$
P_{E}^{t,r} \leq (M - 1) \sum_{l_0, l_1} P_{i_0, i_1|T, R}(l_0, l_1|t, r)
$$

$$
\exp\left[-\frac{Q p^2}{4(p + l_0 + l_1)}\right]
$$

$$
\leq (M - 1) \sum_{l_1=0}^{r-l_1} \left(\frac{r}{l_1}\right) \left(\frac{1}{M}\right)^{l_1} \left(1 - \frac{1}{M}\right)^{r-l_1}
$$

$$
\sum_{l_0=0}^{r-l_0} \left(\frac{r - l_1}{l_0}\right) \left(\frac{1}{M - 1}\right)^{l_0} \left(1 - \frac{1}{M - 1}\right)^{r-l_0-l_1}
$$

$$
\exp\left[-\frac{Q p^2}{4(p + l_0 + l_1)}\right]. \quad (19)
$$

It can be noticed that as $Q \to \infty$, $P_{E}^{t,r} = 0$.

4. Numerical results

In this section, we compare the performance of the following three synchronous optical PPM-CDMA systems:

(i) with interference cancellation,
(ii) with interference cancellation and Manchester codes,
(iii) without interference cancellation [1].

The structures of the above three systems are similar, that is by extending the structure of system (iii) we can obtain those of (i) and (ii) [1]. Eqs. (7) – (19) are used in calculation of bit-error rate of system (ii). The corresponding equations for system (i) are provided in the appendix. We have calculated upper bounds of the bit error rates for systems (i) and (ii) and a lower bound of the bit error rate for system (iii).

Figs. 5 and 6 show variations of the bit error rates with $\mu$ defined as the average received photons per nat for the three optical PPM-CDMA systems with various values of $M$. Both $Q$ and $\mu$ are related by the equation $\mu = Q p / \ln M$. The prime number $p$ and the number of simultaneous users $N$ are set as $p = 11$, $N = 90$ and $p = 13$, $N = 140$ in Figs. 5 and 6, respectively. It can be seen that under the Poisson shot
noise model for the receiver photo-detector, systems (i) and (ii) performances are much better than that of (iii) for moderate values of \( \mu \) and \( M \). The improvement is more apparent as \( \mu \) increases. In fact, the bit-error probability for the systems with interference cancellation will reduces to zero if \( \mu \) approaches infinity. However, when \( M \) and \( \mu \) are too small, the system without cancellation performs slightly better than the other two systems. Our numerical results show that as the value of symbol \( M \) increases, the performance of systems (i) and (ii) improves significantly. The results clearly show that using Manchester codes improves the system performance, however, it increases the system bandwidth.

In Fig. 7 for all three systems, we have shown variations of BER upper bounds with the number of users for \( p = 11 \) and \( M = 8 \). In our calculations we have used \( \mu = 100 \) for systems (i) and (ii) and \( \mu = \infty \) for system (iii). It is obvious that very large number of simultaneous users can be accommodated with relatively low bit-error rates for the systems with interference cancellation as long as \( \mu \) and/or \( M \) are large enough. However, this is not true for the system without cancellation as shown in the numerical results. Again, we can clearly see that the system with interference cancellation and Manchester codes performs better than the system without Manchester codes. The improvement is more significant as the number of simultaneous users \( N \) increases.

Fig. 8 shows the performance of the three systems for the case of full load \((N = p^2 - p)\). In this case (see Fig. 8) the system without cancellation has a relatively constant bit-error rate for various values of \( p \), however, it cannot reach the full load and retain reliable transmission.

The two systems with cancellations show good performance when \( p \) is small, however, their performances become unreliable as \( p \) increases. It should be noted that the two systems become reliable when \( \mu \) increases. In fact according to Eq. (18) we can accommodate the entire number of subscribers (full load) for any given \( p \) by properly increasing the value of \( \mu \).

5. Conclusion

In this paper, a synchronous optical PPM-CDMA system with interference estimation and cancellation has been proposed. The correlation properties of prime code sequence have been used to provide an estimate on the multi-user. This estimated interference has been subtracted from the received signal after photo-detection. In addition, we have used Manchester codes to further improve the overall system bit-error rate performance.

We have compared the bit-error probabilities of our proposed systems with each other and with that of the PPM-CDMA system without interference. Our results reveal that the bit-error rates have considerably improved with the adoption of the aforementioned techniques. Furthermore, the system with Manchester codes outperforms the system without Manchester codes. We have also proved theoretically that the bit-error probability of the proposed systems approaches zero as the average photon count increases to infinity. That is, the error probability floors (which distinguish this types of systems) can be completely removed.
Appendix Optical PPM-CDMA with interference cancellation (without using Manchester codes)

The system with interference cancellation but without using Manchester codes is similar to the system shown in Fig. 4. The only difference is in the range of the integrator. For the system using Manchester codes, the spreading sequences only occupy half of the entire chip time, \( T_i \); therefore integration is only performed over the left or right half of the chip time depending on which group it belongs to as described in Section 2. However, for the system without using Manchester codes, integration is performed over the entire chip time. The number of active users \( T \) in the desired group has the same probability distribution as the system with Manchester codes.

In this system, the contribution of multi-user interference is from users in group 2 up to group \( p \) if the desired user is in group 1. Therefore, given that \( T = t \), it is easy to check that the interference vector \( I \) is given by

\[
P_{I|T}(l_0, l_1, \ldots, l_{M-1}|t) = \frac{1}{M^{N-t} l_0! l_1! \cdots l_{M-1}!}. \tag{A.1}
\]

Similar to our proposed system, symbol \( i \) is declared to be the correct one if \( \tilde{Y}_{1,i} \neq \tilde{Y}_{1,j} \), for every \( j \neq i \). The upper bound of the bit-error probability can be derived as follows:

\[
P_b \leq \frac{M}{2(M-1)} \sum_{l=0}^{l_{\text{max}}} P_E^l P_I(t). \tag{A.2}
\]

In our analysis, we assume equally likely data symbol, therefore

\[
P_E^l = \Pr \{ \tilde{Y}_{1,i} \geq \tilde{Y}_{1,0}, \text{ some } j \neq 0 \mid T = t, V_{1,0} = 1 \}
\leq (M - 1) \sum_{l=0}^{l_{\text{max}}} P_{I|T}(l|t) \Pr \{ \tilde{Y}_{1,i} \geq \tilde{Y}_{1,0} \mid T = t, l = l, V_{1,0} = 1 \}
\leq \sum_{l=0}^{l_{\text{max}}} P_{I|T}(l|t) \theta(t, l), \tag{A.3}
\]

where \( \theta(t, l) \leq (M - 1) \exp \left[ -Q \frac{p^2}{4(p + l_0 + l_1)} \right] \) is the same as in Section 3.

Similarly,

\[
P_{E}^l \leq (M - 1) \sum_{l_0,l_1} P_{I|T}(l_0, l_1|t) \exp \left[ -Q \frac{p^2}{4(p + l_0 + l_1)} \right]
\leq (M - 1) \sum_{l_1=0}^{M-l_1} \left( \frac{N - l_1}{M} \right) \left( \frac{1}{M} \right)^{l_1} \left( 1 - \frac{1}{M} \right)^{N-l_1},
\]

\[
\sum_{l_0=0}^{N-t-l_1} \left( \frac{N - l_0 - l_1}{M} \right) \left( \frac{1}{M} \right)^{l_0} \left( 1 - \frac{1}{M} \right)^{N-l_0-l_1} \exp \left[ -Q \frac{p^2}{4(p + l_0 + l_1)} \right].
\]  \tag{A.4}

References