CO-CHANNEL INTERFERENCE CANCELLATION IN OPTICAL SYNCHRONOUS CDMA COMMUNICATION SYSTEMS
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Abstract - Three co-channel interference cancellation techniques for synchronous optical CDMA communication systems are proposed. Modified prime sequence optical codes that exhibit a grouping characteristic are employed. In the first technique the desired user collects primary decisions from receivers of all interfering users and subtracts them, after properly weighting, from its received signal. In the second and third techniques the desired user collects photodetector outputs from users in its same group and subtracts them from its received signal after a proper scaling.

I. INTRODUCTION

CDMA effectively utilizes the wide bandwidth of optical communication medium [1-5]. Synchronous [1-3] and asynchronous [4-5] CDMA have been considered in the literature. It is known [2] that a synchronous CDMA system accommodates a larger number of users and the available number of spreading codes is larger since the same code can be reused with different phases. The maximum number of users in a CDMA system is limited by the maximum tolerable interference, termed as co-channel interference. This noise-like interference is in fact not completely random. If the receiver is able to cancel it, the system performance can be improved.

This paper proposes three interference cancellation techniques for optical synchronous CDMA. We select the modified prime sequence codes developed in [2] to demonstrate the techniques. The first technique is general while the second and third techniques are more tailored for the modified prime sequence codes. The analysis considers a direct-detection optical channel utilizing number-state light field. The background noise is assumed negligible. The paper is organized as follows. The synchronous CDMA system is described and analyzed in section II without interference cancellation. Sections III, IV and V are devoted for the description and analysis of the three cancellation techniques, respectively. Numerical results are given in section VI. Finally the findings and conclusions are given in section VII.

II. OPTICAL SYNCHRONOUS CDMA

The synchronous CDMA system considered in this paper is similar to that in [2]. It employs modified prime sequence optical PN codes. Given a prime number p, p^2 codes of period p^2 are generated. The p^2 codes are grouped in p groups with p codes in each group. Therefore code j belongs to group \((j-1)/p+1\), where \(\lfloor \cdot \rfloor\) is the integral part of \(\cdot\). The cross-correlation function between two chip synchronous codes \(a(m)\) and \(a(m)\) of length \(p^2\) is given by:

\[
C_{ij}(m) = \sum_{n=0}^{p^2-1} a_i(m+n)a_j(m),
\]

where \(n\) is the phase shift between the two codes. For a synchronous system \(n=0\). Denoting \(C_{ij}(0)\) by \(C_{ij}\), the modified prime sequence codes given in [2] have:

\[
C_{ij} = \begin{cases} 
0 & \text{if } i = j \\
1 & \text{if } i \text{ and } j \text{ are from same group and } i \neq j, \\
1 & \text{if } i \text{ and } j \text{ are from different groups}
\end{cases}
\]

which signifies that at the sampling instant users in the same group will have zero mutual interference while any two users from different groups will have the same mutual interference with unity cross-correlation. Each user in the system is permanently assigned a unique code as its signature. There are \(N\) users, \(N \geq 2\). Codes are assigned to users randomly, with a uniform distribution. Throughout the paper when referring to user \(j\) then reference is also made to code \(j\) and the user is considered in the group that includes code \(j\). A binary on-off keying scheme is used where each user transmits its signature sequence of \(1\)s up to a data bit is \(1\) and ceases transmission otherwise.

All transmitters use the same power so that all signals are received with the same power. The data bits of user \(j\) \(b_j\) are modeled as independent random variables \(\{x_i\}\) that take the values of \(1\) or \(0\) with equal probability. Define the r.v. \(Y_j\) \(\in \{0,1,\ldots,p^2\}^2\) as:

\[
Y_j = \begin{cases} 
1 & \text{if code } j \text{ is assigned to a user} \\
0 & \text{if code } j \text{ is not assigned to a user}
\end{cases}
\]

Therefore \(\sum_{j=1}^{N} Y_j = N\). Without loss of generality for the remainder of this paper we assume that code 1 is assigned to user 1 (\(Y_1 = 1\)) and we evaluate the probability of bit error for user 1. We assume that any receiver knows all the current users, the used codes and their respective groups. The receiver of user 1 correlates the compound received sequence of laser pulses with its address code. The correlator consists of tapped delay lines as explained in [1]. Hence, the correlator photon count of user 1 is proportional to (we will always assume a unity factor of proportionality):

\[
Z_1 = p_{11} + \sum_{j=1}^{N} b_jC_{1j}Y_j = p_{11} + \sum_{j=1}^{N} b_jY_j,
\]

where the interference \(Y\) is equal to the total number of users sending '1' in groups 2, 3, ..., and \(p\). Let the r.v. \(T\) represent the number of users in the first group. The probability density function (pdf) of \(T\), conditioned on \(T\), is therefore given by:

\[
Pr(T = t | T = t) = \frac{1}{c(t)(N-t)} \binom{N-t}{t},
\]

where \(c(t) = p_{11}(N-1)-\sum_{j=2}^{N} b_j\) is the number of users in the first group. The probability of bit error depends on \(N\) and \(T\). Since \(b_1 = 1\) with equal probability, and due to the symmetry of the binomial pdf (7) around its mean, the optimum value of \(\theta\) is half way between the statistics means of \(Z_0b_0=0\) and \(Z_1b_1=1\), giving \(\theta = (N - p - T)/2\) and the probability of error, conditioned on \(T\), is given by
Pr(Ε | T = t) = \frac{1}{2} \left[ Pr(Y > θ | T = t) + Pr(Y < θ - ρ | T = t) \right].

This probability is different for integer or non-integer θ. Using (7) it can be shown that if θ is an integer the probability of error is given by

Pr(Ε | T = t) = \frac{1}{2\pi} \left[ \sum_{i=0}^{N-1} \left( N - i \right) - \frac{1}{2} \left( N - r \right) \right],

(6a)

while if θ is a non-integer the probability of error is given by

Pr(Ε | T = t) = \frac{1}{2\pi} \left[ \sum_{i=0}^{N-1} \left( N - i \right) \right].

(6b)

where r is given by \( r = (N - p - i) / 2 \). If \( p > N + r \) \( r \) \( \leq 0 \) and (6a) and (6b) yield a zero probability of error. Since the minimum value of \( i \) is 1, error free transmission occurs for \( N < p + 1 \).

The probability of error is given by averaging (6) over the pdf of \( T \). Given that user 1 always exists, and with uniform distribution of the \( N - 1 \) other users over the other \( p - 1 \) codes in the system, the pdf of \( T \) is found as

Pr(T = t) = \left( \frac{p^{N-p-1} t^{p-1}}{N-1} \right), \quad t \in \{t_{\text{min}}, \ldots, t_{\text{max}}\}

(7)

where \( t_{\text{min}} = \max \{1, N - (p - p - 1)\} \) and \( t_{\text{max}} = \min \{N, p\} \), where max(a,b) and min(a,b) are the maximum and minimum of a and b, respectively. The optimum probability of error is evaluated from (6) and (7).

III. FIRST TYPE OF CANCELLATION (CANCELLE 1)

Canceller 1 is shown in figure 1. All receivers make a primary decision on whether the transmitted signal is “1” or “0” by comparing the output of the photodetector to a threshold. This threshold depends on \( N \) and the number of users in the group and is the same for users in the same group. Define \( T_1, X_i, Y_i, \) and \( θ_0 \) to be the number of users, the number of users sending “1”, the interference that a receiver gets, and the threshold employed in decision, respectively, for group i. However, since reference to group 1 is very frequent in this paper, in future sections we denote \( T_1, X_1, Y_1, \) and \( θ_0 \) by \( T, X, Y \) and \( θ \), respectively. Clearly there are \( T \) users in group i sending “0” and \( N \) is equal to the sum of all the \( T_i \).

Receiver 1 multiplies the primary decision of user j, for all \( j \neq 1 \), by the cross-correlation \( C_{ij} \) of (2) and adds up all the results. The total is subtracted from \( Z_j \) of (5) to form a new decision variable \( Z \). Equation (2) leads to the implementation shown in figure 1. \( Z_i \) is compared to another threshold \( α \) and “1” is declared to be sent if \( Z_i > α \), otherwise “0” is declared.

We first find the pdfs of \( T_1 \) and \( X_i, (T_i) \) are dependent r.v’s since, conditioned on all \( T_i, (X_i) \) are independent. The pdf of \( T_1 \) is given by (7). In a similar manner the pdf \( Pr(T_1 | T_{i=1}, T_{i=2}, \ldots, T_{i=p}) \) for \( i=2,3,\ldots,p \) is given by:

Pr(T_1 | T_{i=1}, T_{i=2}, \ldots, T_{i=p} = t_{i=1}, \ldots, T_{i=p} = t_i) = \begin{cases} \frac{p(p-i)}{N - \sum_{j=i+1}^{p} t_j} & \text{for } i, \sum_{j=i+1}^{p} t_j \leq N - p \\ \frac{p(p-i)}{N - \sum_{j=i+1}^{p} t_j} & \text{for } i, \sum_{j=i+1}^{p} t_j > N - p \end{cases}

(8)

where \( t_{\text{max}} = \max \{0, N - p(p-i) - \sum_{j=i+1}^{p} t_j\} \), and \( t_{\text{min}} = \min \{p, N - \sum_{j=i+1}^{p} t_j\} \). The pdf’s of \( X_i, (T_i) \), conditioned on \( (T_{i=1}) \), \( i=1,2,\ldots,p \), are given by the binomial distributions, for \( i=1 \):

Pr(X_i = x_i | T_i = t_i, θ_0 = 0) = \left( \frac{1}{2} \right)^{t_i} \binom{t_i}{x_i},

(9a)

where in (9a) \( x_i = 0,1,\ldots,t_i \).

Pr(X_i = x_i | T_i = t_i, θ_0 = 1) = \left( \frac{1}{2} \right)^{t_i} \binom{t_i}{x_i - 1},

(9b)

where in (9b) \( x_i = 0,1,\ldots,t_i \) and for \( i < 1 \) is:

Pr(X_i = x_i | T_i = t_i) = \left( \frac{1}{2} \right)^{t_i} \binom{t_i}{x_i}, \quad x_i = 0,1,\ldots,t_i.

(9c)

For users in any group \( i=1 \) there are \( X_i \) users whose received photon count is \( p \cdot Y_i \), and \( T_i \) users whose received photon count is \( Y_i \), where \( Y_i = \sum_{j=i+1}^{p} X_j \). To generate the primary decision, each receiver applies its photon count as the argument of the function \( φ(a) = \left\{ \begin{array}{ll} 1 & a ≥ θ \end{array} \right. \). The receiver of user 1 builds the new decision r.v.

\[ Z_i = Z_i - \sum_{j=i+1}^{p} X_j \cdot φ(p + Y_j) + (T_i - X_i) \cdot φ(Y_i) \]

\[ = φ(a) + \sum_{j=i+1}^{p} X_j [1 - φ(p + Y_j) + φ(Y_i) - T_i \cdot φ(Y_i)] \]

(10)

\[ = \phi(a) + C_{ij} \]

To decide on the transmitted bit, \( Z_i \) is applied to the function \( p(\alpha) = \left\{ \begin{array}{ll} 1 & a ≥ \alpha \end{array} \right. \). The probability of error is given by

\[ P_e = \frac{1}{2} Pr[E|E = 0] + Pr(E|E = 1) \]

(11)

Let the vectors \( T = (T_1, T_2, \ldots, T_p) \) and \( X = (X_1, X_2, \ldots, X_p) \). Then

\[ Pr(E|E = 0, T = t, \bar{X} = x) = Pr(\bar{Z}_i > α) \]

(12)

where (12) derives from the fact that, given \( T \) and \( \bar{X} \), \( Z_i \) in (12) is deterministic. Averaging (12) over \( \bar{X} \) and then over \( T \), \( T_{p-1}, T_{p-2}, \ldots, \) and then \( T_1 \) with that order, one can find:
\[ \Pr(Z|b_i = 0) = \frac{1}{2} \sum_{i=1}^{H} \sum_{j=1}^{T_i} \sum_{k=0}^{N_p-1} \sum_{l=0}^{N_p-1} \cdots \sum_{l=0}^{N_p-1} \]
\[ \times A_{i,j,k,l} \times A_{i,j,k,l} \times B_{i,j,k,l} \times B_{i,j,k,l} \times p(G_i) \]

where \( G_i \) is defined in (10) and
\[ A_i = \left( \binom{i-1}{h_i-1} \right) \] and \( B_i = \left( \binom{i}{t_i} \right) \) (14.a)
\[ B_i = \frac{p^2 - p + 1 - p}{N - p} \] and \( B_i = \frac{p^i}{N - 1} \) (14.b)

Similarly one can find (note the difference in the limits of \( x_i \))
\[ \Pr(Z|b_i = 1) = \frac{1}{2} \sum_{i=1}^{H} \sum_{j=1}^{T_i} \sum_{k=0}^{N_p-1} \sum_{l=0}^{N_p-1} \cdots \sum_{l=0}^{N_p-1} \]
\[ \times A_{i,j,k,l} \times A_{i,j,k,l} \times B_{i,j,k,l} \times B_{i,j,k,l} \times [1 - q(p + G_i)] \]

Using (11), (13) and (15) one can find the probability of error for canceller 1. Numerical search for the optimum thresholds \( \theta_i \) and \( \alpha \) is needed. It is found that the combination of any \( \alpha \) that satisfies 0 < \( \alpha < 1 \) and \( \theta_i = \lfloor (N + p - 1)/2 \rfloor \) consistently provide an optimum or sufficiently close to optimum performance for all values of \( N \) and \( p \).

IV. SECOND TYPE OF CANCELLATION (CANCELLER 2)

From (2), users in the same group get the same interference. The receiver of one user can employ the photodetector output of other receivers in its group to cancel its own interference. A straight forward approach is to reserve one code in each group from being assigned to any user. The photodetector output of the correlator of this code consists only of the interference this group gets, which can then be subtracted from all other photodetector outputs. The cost for such system is the loss of \( p \) codes, one in each group.

Canceller 2 retains the advantage of error free reception for any group with one or more codes not assigned. It also provides interference cancellation when the codes are assigned to users randomly and when the system is full loaded (\( N = p^2 \)).

Figure 2 shows canceller 2 with the constant \( k = 1 \) in the figure. Considering group 1, the received signal is correlated with all codes in the same group, i.e., \( p \) correlators are employed even if the number of users in the group < \( p \). User 1 collects photodetector outputs of all the \( p \) receivers. Similar to (4) these outputs are:
\[ Z_i = p b_i y_i + Y_i \quad i \in \{1, 2, \ldots, p\} \] (16)
The first receiver forms a new decision variable \( Z_i \):
\[ Z_i = (p - 1)Z_i - \sum_{j=2}^{p} Z_j \]
\[ = (p - 1)(p b_i Y_i + Y_i) - Y_i - (X - b_i) p \] (17)

The pdf of \( X \) is given by (9a and 9b). The receiver declares that “1” is sent if \( Z_i > \theta \) and “0” otherwise. The optimum value of \( \theta \) is half way between the statistical means of \( Z_i|b_i = 0 \) and \( Z_i|b_i = 1 \), which is \( \theta = \frac{1}{2}((p^2 - p - T) \). By a simple sketch of the pdf’s of \( Z_i|b_i = 0 \) and \( Z_i|b_i = 1 \) one can find that the probability of error vanishes if \( T = p \) (two pdf’s do not overlap). Therefor error occurs only if \( T > p \) and the optimum threshold \( \theta = 0 \), independent of the number of users in the group and the total number of users. \( Z_i|b_i = 0 \) is always \( \leq 0 \) since \( X \geq 0 \) and hence the probability of error is zero for \( b_i = 0 \). For \( b_i = 1 \) we have \( \Pr(E|b_i = 1, T = \theta) = \Pr(X > \theta) \), and from (9b) we get
\[ \Pr(E|b_i = 1, T = \theta) = \begin{cases} \frac{1}{2} & \text{if } T = p \\ 0 & \text{if } T < p \end{cases} \] (18)

As noted above, (18) signifies that if any group has a code not assigned to a user the remaining users in this group enjoy error free reception. The average probability of error is then given from (7) and (18) as
\[ P_e = \begin{cases} 0 & \text{if } N < p \\ \frac{p^2 - p}{2p} \cdot \frac{N - p}{N - 1} & \text{if } N \geq p \end{cases} \] (19)

V. THIRD TYPE OF CANCELLATION (CANCELLER 3)

In canceller 2 the interference term \( Y \) in (16) is completely removed and another interference \( X \) arises. Canceller 3, on the other hand, removes \( Y \) only partially. Canceller 3 is shown in figure 2 with some constant \( 0 < k < 1 \).

Instead of (7) canceller 3 forms the decision variable \( \tilde{Z}_i \) as:
\[ \tilde{Z}_i = (p - 1)Z_i - k \sum_{j=2}^{p} Z_j \]
\[ = p(p - 1)h_i b_i (X - b_i) + kp(X - b_i) Y \] (20)

Receivers 1 declares that “1” is sent if \( \tilde{Z}_i > \theta \) and “0” otherwise. From (20) since \( X \) and \( Y \) have the independent binomial distributions (9a and 9b) and \( (X - b_i) \), respectively, it can be shown that the optimum threshold for group 3 is given by:
\[ \theta = \left( (N - T + p)(1 - k) + kp(1 - k) \right) / 2 \]

We have
\[ \Pr(E|b_i = 0, T = \theta) = \Pr(\tilde{Z}_i > \theta) \]
\[ = \Pr \left( Y > \frac{kp}{(p - 1)(1 - k)} \frac{X}{(p - 1)(1 - k)} + \frac{\theta}{(p - 1)(1 - k)} \right) \] (21)

and
\[ \Pr(E|b_i = 1, T = \theta) = \Pr(\tilde{Z}_i \leq \theta) \]
\[ = \Pr \left( Y < \frac{kp}{(p - 1)(1 - k)} \frac{X}{(p - 1)(1 - k)} + \frac{\theta}{(p - 1)(1 - k)} \right) \] (22)

The value of \( k \) is selected to be \( k = \frac{1}{2} \) and it is readily shown that it provides the optimum performance. However, there could be other values that provide optimum performance.
as well. From (20) the pdf of \( Z \) is \( N \leq 0 \) exists only over the
interval \((kp(T - 1), (p - 1)(1 - k)(N - T))\). Also the pdf of \( Z \) is \( N \geq 1 \) exists only over the interval \((kp(T - 1), (p - 1)(1 - k)(N - T))\). Errors can exist only if these two pdf’s overlap.

After substituting with the value of \( k \) we find that error free transmission occurs if:

\[
(p - 1)(N - T) < k(p - 1)(1 + p^2) - p^2(T - 1) \tag{23}
\]

At this point two cases arise:

**Case 1: T < p**

Maximizing the LHS of (23) by letting \( N - T \) to be \( N - 1 \) and \( p - 1 \) to be \( p \) and also minimizing the RHS by letting \( T \) to be \( p \), the condition (23) becomes \( N < p^2 + p \). Hence, for \( T = p \), canceller 3 provides error free transmission. Hence, canceller 3 retains the advantage of error free reception for any group with a code not being assigned to a user.

**Case 2: T = p**

From (23) error free transmission occurs if \( N < 2p \). For (21) we have

\[
Pr(\varepsilon| h = 0, T = p) = \Pr\left( Y > \frac{kp}{p - 1}(1 - k) X + \frac{N}{2} \right) \tag{24}
\]

averaging (24) over \( X \), using (9,a) we get

\[
Pr(\varepsilon| h = 0, T = p) = \Pr(X = 0)Pr\left( Y > \frac{N}{2} \right) \tag{25}
\]

The second term in the RHS of (25) is equal to zero. Applying (9,a) and (7), (25) yields

\[
P(\varepsilon| h = 0, T = p) = \frac{1}{2^{N-p}} \sum_{x=1}^{N-p} (N - p - y) \tag{26}
\]

In a similar manner, from (22)

\[
Pr(\varepsilon| h = 1, T = p) = \Pr\left( Y \leq \frac{kp(X - 1)}{(p - 1)(1 - k)} - \frac{N}{2} \right) \tag{27}
\]

and averaging over \( X \), we get

\[
Pr(\varepsilon| h = 1, T = p) = \Pr(X = p) Pr\left( Y \leq \frac{N}{2} \right) \tag{28}
\]

Again the second term in the RHS of (28) vanishes and similar to (26) we get

\[
P(\varepsilon| h = 1, T = p) = \frac{1}{2^{N-p}} \sum_{x=1}^{N-p} (N - p - y) \tag{29}
\]

We recall that the choice of \( k \) given earlier is optimum. For \( T = p \) this choice yields a zero probability of error. For \( T < p \) we note in (25) and (28) that the probability of error is also minimized. Finally from (26) and (29) the probability of error is given by

\[
P_e = \left\{ \begin{array}{ll} 
\frac{1}{2^{N-p}} \sum_{x=1}^{N-p} (N - p - y) & N \text{ odd} \\
\frac{1}{2^{N-p}} \sum_{x=1}^{N-p} (N - p - y) + \frac{1}{2} & N \text{ even}
\end{array} \right.
\]

**VI. NUMERICAL RESULTS AND DISCUSSION**

Figures 3 and 4 compare the performance of all systems for all \( p = 5 \) and 11, respectively. The performance of the non-cancellation system is very poor when the number of users is a multiple of \( p \). Cancellation 1 improves the performance when the number of users is low. With larger number of users its performance is even worse than the non-cancellation system. Cancellers 2 and 3, however, provides a significant improvement over both the non-cancellation system and canceller 1 for \( p = 5 \) and 11. Cancellation 3 outperforms all systems for all range of number of users. Figure 5 compares the performance of cancellers 2, 3 and the non-cancellation system for higher values of \( p \). Both the cancellers demonstrate a significant improvement that increases rapidly as \( p \) increases.

Canceller 3 offers a consistent improvement over canceller 2 for the range of \( p \) and \( N \) shown. Figure 6 shows the performance in case of full load, i.e., \( N = p^2 \). Again both cancellers offer a significant improvement over the non-cancellation system. Cancellation 3 continues to out-perform canceller 2 at this high load.

**VII. CONCLUSION**

Three interference cancellation techniques have been proposed for synchronous optical CDMA systems. In the first, the desired user collects primary decisions from all interfering users and subtracts them from its signal. In the second and third, the desired user collects all other photodetector outputs in its own group and subtracts them from a scaled version of its received signal. The average probability of error is evaluated and the results are compared. A significant improvement of the cancellation technique over the system without cancellation. C canceller 1 is effective when the number of users is low. Cancellers 2 and 3 provide a significant improvement. Cancellation 3 outperforms canceller 2.

**REFERENCES**


Figure 1, Optical CDMA System with Cancellor 1

Figure 2, Optical CDMA System with Cancellor 2 and Cancellor 3

Figure 3, Probability of error versus the number of users with \( p = 5 \)

Figure 4, Probability of error versus the number of users with \( p = 11 \)

Figure 5, Probability of error versus the prime number with \( N = 2p, N = 5p \)

Figure 6, Probability of error versus the prime number with \( N = p^2 \)